MAY 2024 **U/ID 32505/UCM2B**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Prove that the Tangents at the ends of a pair of conjugate diameters of an ellipse form a parallelogram of constant area.

2. Prove that the sum of the squares of two conjugate semi diameters of an ellipse is constant.

3. Find the asymptotes of the hyperbola equation.

4. Find the equation of the circle which has its centre at the point (6, 1) and touches the straight line $5x + 12y = 3$.

5. Prove that the equation $2x^2 - 6y^2 - 12z^2 + 18yz + 2zx + xy = 0$ represents a pair of planes.

6. Obtain the equation of the plane through the point (x, y, z) and parallel to the planes $ax + by + cz + d = 0$.

7. Prove that the equations to the line through (α, β, γ) at right angles to the lines.

$$\frac{x}{l_1} = \frac{y}{m_1} = \frac{z}{n_1}; \frac{x}{l_2} = \frac{y}{m_2} = \frac{z}{n_2} \text{ are}$$

$$\frac{x-\alpha}{m_1 n_2 - m_2 n_1} = \frac{y-\beta}{n_1 l_2 - n_2 l_1} = \frac{z-\gamma}{l_1 m_2 - l_2 m_1}.$$

8. Examine the nature of intersection of the plane $2x - 5y + z = 3;\ x + y + 4z = 5;\ x + 3y + 6z = 1$.

9. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z = 0$ at (1, 2, 3).

10. Find the equation of the sphere whose centre is (–6, 1, 3) and radius 4.

11. Define orthogonal projection on a plane.

12. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9, 2x + 3y + 4z = 5$ and the point (1, 2, 3).

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touches the circle $x^2 + y^2 = r^2$. Find the locus of their poles.

14. Find the angle between the lines $r\cos(\theta \cdot \theta_1) = P_1$ and $r\cos(\theta \cdot \theta_2) = P_2$.

15. A triangle the lengths of whose side are a, b and c is placed so that the middle points of the sides are on the axes. Show that the equation to the plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$, where

$$\alpha^2=\frac{(b^2+c^2-a^2)}{8},\ \beta^2=\frac{(c^2+a^2-b^2)}{8},$$

$$\gamma^2=\frac{(a^2+b^2-c^2)}{8}.$$

16. Find the shortest distance between the axis of z and the line.

$$ax+by+cz+d=0,\ a'x+b'y+c'z+d'=0.$$

17. Find the equation of the sphere that passes through the circle

$$x^2+y^2+z^2-2x+3y-4z+6=0$$
$$3x-4y+5z-15=0$$

and cuts the sphere $x^2+y^2+z^2+2x+4y-6z+11=0$ orthogonally.

18. Find the centre and the radius of the sphere $x^2+y^2+z^2+2x-4y+2z-3=0$.

19. Find the co-ordinates of the point of intersection of tangents at the points where $x+4y=14$ meets the circle $x^2+y^2-2x+3y=5$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that the acute angle between two conjugate diameters of an ellipse is minimum when they are equal.

21. Find the equation of the circle on the line joining the points (r_1, θ_1) and (r_2, θ_2) as diameter.

22. Show that the origin lies in the acute angle between the planes $x + 2y + 2z = 9$ and $4x - 3y + 12z + 13 = 0$. Find the planes bisecting the angles between them and point out which bisects the acute angle.

23. Find the magnitude and the equation of the line of the shortest distance between the lines

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

24. Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$.