MAY 2025 **U/ID 32505/UCM2B**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Find the pole of the line $lx + my + n = 0$ with respect to the circle $x^2 + y^2 = a^2$.

2. What is the condition for the lines $lx + my + n = 0$ $l_1x + m_1y + n_1 = 0$ to be longitude with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

3. Define conjugate diameters of the hyperbola.

4. Find the locus of the tool of the perpendiculars drawn from the pole to the tangents to the circle $r = 2a\cos\theta$.

5. Find the equation to the asymptotes of $2x^2 + 2xy - 3x + y = 0$.

6. Find the equation of the plane through the point $(1, 3, 4)$ and parallel to the plane $2x + 3y - 4z + 8 = 0$.

7. Find the angle between the planes $2x + 2y + z = 5$, $3x - 2y - z = 6$.

8. Find the distance of the origin from the plane $6x - 3y + 2z - 14 = 0$.

9. Find the equation of the straight line joining the points $(2, 3, 7)$ and $(2, -5, 8)$.

10. Find the symmetric form of the line

$3x - 2y + z - 1 = 0 = 5x + 4y - 6z - 2$.

11. Find the coordinate of the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z - 15 = 0$.

12. Find the equation of the sphere which has its centre at the point (6, –1, 2) and touches the plane $2x - y + 2z - 2 = 0$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Find the coordinate of the point of intersection of tangents at the points where $x + 4y = 14$ meets the circle.

14. Find the eccentricity of the ellipse if $y = x$ and $3y = -2x$ are a pair of its conjugate diameters.

15. Prove that the area of the triangle formed by the asymptotes and any tangent to a hyperbola is a constant.

16. Find the equation of the plane passing through the points $(2, -5, -3)$, $(-2, -3, 5)$ and $(5, 3, -3)$.

17. Show that the origin lies in the acute angle between the planes $x + 2y + 2z = 9$, $4x - 3y + 12z + 13 = 0$. Find the planes bisecting the angles between them and point out which bisects the obtuse angle.

18. Find the image of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane $3x - 3y + 10z - 26 = 0$.

19. Find the equation of the sphere through the four points (2, 3, 1), (5, –1, 2), (4, 3, –1) and (2, 5, 3).

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Trace the curve $\frac{12}{r} = 4 + \sqrt{3}\cos\theta + \sin\theta$.

21. Find the centre, eccentricity, foci and directories of the hyperbola.

22. A plane meets the coordinate axes at A, B, C such that the centroid of the ΔABC is the point (α, β, γ). Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$.

23. A line of constant length has its extremities on two given straight line. Find the equation of the surface generated by it.

24. The plane ABC, whose equation is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B, C. Find the equation to the circumcircle of the triangle ABC and obtain the co-ordinate of it centre and radius.

———————