DECEMBER 2022 **U/ID 32507/UCM3A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x \, dx$.

2. Evaluate $\int \sec^3 x \, dx$.

3. State Bernoulli's formula.

4. Write down double integral interms of polar coordinates.

5. Define triple integral.

6. Find the value of $\frac{\partial(x, y)}{\partial(r, \theta)}$.

7. Evaluate $\int_0^{\infty} e^{-x} dx$.

8. Find the value of $\overline{|(1)}$.

9. Define gradient of a secular function.

10. Find the value of $div.(f \times g)$

11. State Green's theorem in a plane.

12. Evaluate $\int_c \vec{F} \cdot d\vec{r}$ if $\vec{F} = 3xy\,\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curves $x = t^2$, $y = 2t^3$, $z = t^3$ from $t = 0$ to $t = 1$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Prove that $\int e^{2x} \cos 3x\, dx = \frac{e^{2x}}{13}(2\cos 3x + 3\sin 3x)$.

14. Using Bernoulli's formula, Evaluate $\int x^4 e^x dx$.

15. Evaluate $\int_0^a \int_0^b (x^2 + y^2)\, dx\, dy$.

16. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

17. Define recurrence formula for gamma function.

18. Find the unit normal to $xy = z^2$ at (1, 9, – 3)

19. If $\vec{A} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$ evaluate $\int_c A \cdot d\vec{r}$.

Where C is the curve $y = x^3$ in the xy - plane from the point (1, 1) to (2, 8).

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. If $I_n = \int \cos ec^n x dx$, prove that $(n-1) I_n = -\cos ec^{n-2} x \cot x + (n-2) I_{n-2}$.

21. Find the area of the surface of the sphere of radius '*r*'.

22. Derive relation between Beta and gamma functions.

23. Evaluate $\iint_s x^3 dydz + x^2 ydzdx + x^2 z\, dxdy$ where S is the surface of the cube $x = 0, y = 0, z = 0$, $x = 1$, $y = 1$, $z = 1$.

24. Find the common area between $y^2 = 4x$ and $x^2 = 4y$ by using Green's theorem.

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