DECEMBER 2023 **U/ID 32507/UCM3A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions

1. Evaluate $\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$.

2. Evaluate $\int_0^{\frac{\pi}{4}} \tan^3 x\,dx$.

3. Find the centroid of the area enclosed by the parabola $y^2 = 4ax$, the axis of x and the latus rectum of the parabola.

4. Find the volume of a segment of height h of a sphere of radius a.

5. Evaluate $\int_0^{\infty} \frac{dx}{a^2 + x^2}$.

6. Define Gamma function.

7. If ϕ is a scalar point function then prove that the divergence of the gradient of ϕ is $\nabla(\nabla\phi) = \nabla^2\phi$.

8. Show that $\overline{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational.

9. Evaluate $\int_0^a \int_x^a (x^2 + y^2)\, dy\, dx$.

10. Evaluate $\int_0^1 \int_0^1 dx\, dy$

11. Evaluate $\int \operatorname{cosec}^4 x\, dx$.

12. State Green's theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions

13. Evaluate $\int x^n \cos ax\, dx$, where n is a positive integer.

14. By changing into polar co-ordinates evaluate the integral $\int_0^{2a\sqrt{2ax-x^2}} \int_0^{} (x^2 + y^2)\, dxdy$.

15. Evaluate $\int_{-1}^{-1} \frac{dx}{x^{2/3}}$.

16. If $V = W \times r$, where W is a constant vector and $r = xi + yi + zk$, then show that $\frac{1}{2}\text{curl } V = W$.

17. Evaluate the integral $I = \int_c x\,dx + y\,dy + z\,dz$.

18. Evaluate $\iiint_V \nabla.F\ dv$. If $F = x^2 i + y^2 j + z^2 k$ and if V is the volume of the region enclosed by the cube $0 \le x, y, z \le 1$.

19. If $F = x^3 i + y^3 j - z^3 k$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. Show that $\iint_S F \cdot ds = \frac{12}{5}\pi a^5$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

20. Evaluate the integral $\mathrm{Im}, n = \int x^m (\log x)^n dx$. where m, n are positive integers.

21. Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

22. Find the relation between Beta and Gamma function.

23. Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$. Also, show that if $\nabla^2 f(r) = 0$, then $f(r) = \frac{\alpha}{r} + \beta$. where α and β are arbitrary constants.

24. State and prove Green's theorem.
