MAY 2023 **U/ID 32507/UCM3A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Evaluate $\int_0^{\frac{\pi}{2}} \cos^8 x dx$

2. Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^5 x dx$

3. Evaluate $\int \tan^4 x dx$.

4. Define double integral.

5. Write down the formula for total volume of solids of revoluation.

6. Define Jacobian for two variables.

7. Evaluate $\int_1^{\infty} \frac{dx}{x^2}$

8. Define Gamma function.

9. Define divergence of vector point function.

10. Define irrotational vector.

11. State Stoke's theorem.

12. Find curl $\vec{F}$ if $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Find the reduction formula for $\int_a^a x^n e^{-x} dx$.

14. Evaluate $\int x^3 \cos 2x \, dx$

15. Evaluate $\iint (x^2 + y^2) dx \, dy$ over the region for which x, y are each ≥ 0 $x + y \leq 1$.

16. Find the area of the curvilinear bounded by the four parabolas $y^2 = ax, y^2 = bx, x^2 = cy, x^2 = dy$.

17. Prove that $\left| \frac{1}{2} = \sqrt{\pi} \right.$

18. Evaluate by Green's theorem in the plane for $\oint_C \left[\left(x^2+y^2\right)dx-2xydy\right]$ where C is the rectangle bounded by $y=0, x=0, y=b, x=a$.

19. Prove that $\iint_S \vec{r}.ds=3V$ where S is any closed surface enclosing a volume V.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Find the reduction formula for $\int x^m(\log x)^n dx$ and evaluate $\int x^4(\log x)^3 dx$.

21. Evaluate $\iiint xyzdxdydz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$.

22. Express $\int_0^1 x^m\left(1-x^n\right)^p dx$ interms of gamma functions and evaluate the integral $\int_0^1 x^5\left(1-x^3\right)^{10} dx$.

23. Prove that $r^n\vec{v}$ is irrotational find 'n' when it is solenoidal.

24. Use Gauss divergence theorem to find $\iint_S \vec{F}.\vec{n}ds$ where $\vec{F}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$ where S is the surface of the sphere $x^2+y^2+z^2=a^2$.