MAY 2024 **U/ID 32507/UCM3A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find the value of $\int x^n \, dx$.

2. Find the value of $\int_0^{\frac{\pi}{2}} \sin^7 x \, dx$.

3. Define double integral.

4. Evaluate $\int_0^2 \int_0^2 dx \, dy$ taken over the positive quadrant.

5. Evaluate $\int_0 \frac{dx}{x}$.

6. Define Beta function.

7. If A and B are vector point function. Show that $\nabla(A \times B) = (\nabla \times A) \cdot B - (\nabla \times B) \cdot A$.

8. Show that $\nabla^2 r^n = n(n+1)r^{n-2}$ where n is a constant.

9. Define line integral.

10. Define surface integral.

11. Define volume integral.

12. If a is a constant vector. Show that $\nabla \cdot (a \times r) = 0$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Evaluate $\int x^3 \cos^2 x \, dx$.

14. Evaluate $\iint (x^2 + y^2) dx \, dy$. Over the region for which x, y are each ≥ 0 and $x + y \leq 1$.

15. Consider $\int_0^1 \frac{dx}{\sqrt{1 - x^2}}$.

16. The directional derivative of ϕ in the direction specified by the unit vector e is $(\nabla \phi) \cdot e$.

17. If $\vec{F} = xz\,\vec{i} + yz\,\vec{j} + z^2\vec{k}$ evaluate $\int_C \vec{F} \cdot d\vec{r}$ from the point (0, 0, 0) to (1, 1, 1).

18. Evaluate $\int_C x\,dx + y\,dy$, where C is the ellipse $x^2 + 4y^2 = 4$.

19. Show that $\iint_S r \cdot n\,ds = 4na^3$ if S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Evaluate the integral $I_{m,n} = \int \sin^m x \cos^n x\,dx$.

21. Find the centroid of a loop of the lemniscate $r^2 = a^2 \cos 2\theta$.

22. State some properties of Beta function.

23. If ϕ is a scalar point function then prove that

(a) divergence of the gradient of ϕ and

(b) curl of the gradient of ϕ vanishes that is $\nabla \times (\nabla \phi) = 0$ if ϕ has continuous partial derivatives.

24. State and prove Gauss divergence theorem.

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