MAY 2025 **U/ID 32507/UCM3A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. State Bernoulli's formula.

2. Evaluate $\int_0^{\pi/2} \cos^5 x \, dx$.

3. Define double integral.

4. Write the change of variables in the case of three variables.

5. Prove that $\int_{-\infty}^{\infty} \frac{dx}{4 + x^2}$ is convergent.

6. Define Gamma function.

7. Prove that $\nabla \cdot \vec{c} = 0$, where c is a constant.

8. If $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$, then find $\nabla \times \vec{F}$.

9. Define surface integral.

10. State Green's theorem.

11. Find $\int_0^{\pi/2} \sin^6 x \cos^4 x \, dx$.

12. What is meant by Jacobian?

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Find the reduction formula for $\int x^n e^{ax} \, dx$.

14. Evaluate $\iint xy \, dx \, dy$ over the region in the positive quadrant for which $x + y = 1$.

15. Prove that $\lceil(n+1) = \lceil(n)$.

16. Find a unit vector normal to the surface $x^2 - y^2 + z = 2$ at the point $(1, -1, 2)$.

17. Evaluate by Stoke's theorem,

$\int_C \left(e^x dx + 2y\,dy - dz\right)$ where C is the Curve $x^2 + y^2 = 4,\ z = 0, z = 2$.

18. Prove that $div\,(grad\,\phi) = \nabla^2\phi$.

19. Evaluate $\int_0^{\pi/2} x\left(1 - x^2\right)^{1/2} dx$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Obtain the reduction formula for $\int \sin^n x\,dx$, n being a positive integer.

21. Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xy\,dx\,dy$ and evaluate it.

22. Prove that $\beta(m,n) = \dfrac{\lceil m \lceil n}{\lceil m+n}$.

23. If $\vec{F} = x^2 y\,\vec{i} + y^2 z\,\vec{j} + z^2 x\,\vec{k}$, then find Curl Curl $\vec{F}$.

24. Verify the Gauss divergence theorem for $\vec{F} = 4x\,z\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1, z = 0, z = 1$.