DECEMBER 2023 **U/ID 32508/UCM3B**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Verify that the equation $(y - x^3)\,dx + (x + y^3)dy = 0$ is exact.

2. Solve $y^1 = (x + y)^2$.

3. Find the integrating factor of $y^1 + \frac{y}{x} = 3x$.

4. Show that e^x and e^{-x} are linear independent solutions of $y'' - y = 0$ on any interval.

5. Verify that $y_1 = x$ is a solution of $x^2y'' + xy' - y = 0$.

6. Write Lipschitz's condition.

7. Find the exact solution of the initial value problem $y^1 = y^2,\ y(0) = 1$ starting with $y_0(x) = 1$, to calculate $y_1(x)$.

8. Form the partial differential equation from the following by eliminating the constants $z = (x^2 + a)(y^2 + b)$.

9. Write Lagrange's partial differential equation of first order.

10. Define integral surface.

11. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x$.

12. Solve $pq = 1$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Verify that $(x + y)dx - (x - y)\,dy = 0$ is a homogeneous equation and solve is.

14. Solve the equation by finding an integrating factor $(xy - 1)\,dx + (x^2 - xy)\,dy = 0$

15. Prove that if $y_1(x)$ and $y_2(x)$ are any two solutions of $y'' + p(x)y' + Q(x)y = 0$, then $C_1 y_1(x) + C_2 y_2(x)$ is also a solution for any constants C_1 and C_2.

16. If $y_1 = x$ is an obvious solution of the equation $y'' - \frac{x}{x-1}y' + \frac{1}{x-1}y = 0$, find their general solution.

17. Find the exact solution of the initial value problem $y' = 2x(1+y),\ y(0) = 0$ starting with $y_0(x) = 0$. Calculate $y_1(x), y_2(x)\, y_3(x), y_4(x)$ and compare these results with the exact exact solution.

18. Find the general integral of the partial differential equation $(y+zx)p - (x+yz)q = x^2 - y^2$.

19. Find the complete integral of the partial differential equation $p^2z^2 + q^2 = 1$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Find the orthogonal trajectories of the family $y = x + ce^{-x}$.

21. Find the general solution of the equation $y'' + 10y' + 25y = 14e^{-5x}$.

22. State and prove Picard's Theorem.

23. Find the general integral of the linear partial differential equations $pz - qz = z^2 + (x + y)^2$.

24. Find the complete integral of $(p^2 + q^2)y = q^2$ using Charpit's Method.
