MAY 2022 **U/ID 32508/UCM3B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define a homogeneous differential equation.

2. Test the equation $e^y dx + (xe^y + 2y)\,dy = 0$ for exactness.

3. Write the general form of second order linear equation.

4. If $y_1(x)$ and $y_2(x)$ are any two solution of a second order homogenous equation then prove that $c_1 y_1(x) + c_2 y_2(x)$ where c_1 and c_2 are constants is also a solution.

5. Solve $y'' + y' = 0$.

6. If $y_1 = x$ is a solution of $x^2 y'' + xy' - y = 0$, then find the general solution.

7. Find the general solution of $y'' + y' - 6y = 0$.

8. Reduce the second order liner equation $\frac{d^2x}{dt^2} + P(t)\frac{dx}{dt} + Q(t) = 0$ to linear system of equations.

9. Write the trivial solutions of

$$\frac{dx}{dt} = a_1(t)x + b_1(t)y$$

$$\frac{dx}{dt} = a_2(t)x + b_2(t)y\,.$$

10. Write the Lagrange's type partial differential equation of first order.

11. Define integral surface.

12. Write the complete integral of $f(p,q) = 0$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Solve $(x + y)\,dx - (x - y)dy = 0$.

14. Find the integrating factor of $ydx + (x^2y - x)\,dx = 0$.

15. Show that $y = c_1 \sin x + c_2 \cos x$ is the general solution of $y''+y = 0$ on any interval, and find the particular solution for which $y(0) = 2$ and $y'(0) = 3$.

16. Find the particular solution of $y''+y = \sin x$.

17. Solve $\frac{dx}{dt} = x + y$, $\frac{dy}{dt} = 4x - 2y$.

18. Form the partial differential equation by eliminating constant from $z = ax + by + ab$.

19. Find the complete integral of $\sqrt{p} + \sqrt{q} = 1$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. (a) Solve $\frac{dy}{dx} + \frac{1}{x} y = 3x$.

(b) Solve $xy''-y' = 3x^2$.

21. If $y_1(x)$ and $y_2(x)$ are any two solutions of equation $y''+P(x)y'+Q(x)y = 0$ on $[a,b]$, then prove that their wronskian $W = W(y_1, y_2)$ is either identically zero on never zero on [*a, b*].

22. Show that

$$\begin{cases} x = 2e^{4t} \\ y = 3e^{4t} \end{cases} \text{ and } \begin{cases} x = e^{-t} \\ y = e^{-t} \end{cases}$$

are solutions of the homogeneous systems

$\frac{dx}{dt} = x + 2y$, $\frac{dy}{dt} = 3x + 2y$. Also shows that the given solutions are linear independent on every closed interval, and write the general solution of this system.

23. (a) Find the partial differential equation of the family of planes, the sum of whose x, y, z intercepts is equal to unity.

(b) Find the general integral of

$y^2 p - xyq = x(z - 2y)$.

24. (a) Find the complete integral of the pole $p(1 + q) = qz$.

(b) Find the complete integral of $pq = 1$.