MAY 2023 **U/ID 32508/UCM3B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Write the necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact differential equation.

2. Find the integrating factor for

   $ydx + \left(x^2 y - x\right)dy = 0$.

3. If $y_1(x)$ and $y_2(x)$ are any two solutions of $y'' + P(x)y^1 + 2(x)y = 0$, then show that $C_1 y_1(x) + C_2 y_2(x)$ is also a solution for any constants C_1 and C_2.

4. Solve $y'' + y' = 0$.

5. Show that e^x and e^{-x} are linearly independent solutions of $y'' - y = 0$ on any interval.

6. Define linear system of non-homogeneous equations.

7. If the auxiliary equation of the system

$$\frac{dx}{dt} = a_1 x + b_1 y$$
$$\frac{dy}{dt} = a_2 x + b_2 y$$

has distinct real roots, then write the several solution of the system.

8. Find the simple non-trivial solution of

$$\frac{dx}{dt} = x + y, \ \frac{dy}{dt} = 4x - 2y .$$

9. Form the partial differential equation by eliminating a and b from $z = ax + by + a^2 + b^2$.

10. Form the partial differential equation by eliminating the arbitrary function from $z = f\left(x^2 - y^2\right).$

11. Solve : $p^2 + q^2 = 1$.

12. Solve : $z = px + qy + 2\sqrt{pd}$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Solve : $\left(1-x^2\right)\frac{dy}{dx}+2xy=x\sqrt{1-x^2}$ given $y=0$ when $x=0$.

14. Solve $\left(3x^2+6xy^2\right)dx+\left(6x^2y+4y^3\right)dy=0$.

15. Show that $y_1=x$ is a solution of $x^2y''+xy'-y=0$. Also find the general solution.

16. If the homogeneous system

$\frac{dx}{dt}=a_1(t)x+b_1,(t)y;\frac{dy}{dt}=a_2(t)a+b_2(t)y$ has two solutions on $[a,b]$ is

$x=x_1(t)$, $y=y_1(t)$ *and* $x=x_2(t)$, $y=y_2(t)$

then prove that

$x=C_1x_1(t)+C_2x_2(t)$

$y=C_1y_1(t)+C_2y_2(t)$ is also a solution on $[a,b]$ for any constants C_1 and C_2.

17. Find the general integral of the PDE $pz-qz=z^2+(x+y)^2$.

18. Find the complete integral of $\sqrt{p}+\sqrt{q}=1$.

19. Find the complete integral of $p^2+q^2=x+y$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Find the shape of the curved mirror such that light from a source at the origin will be reflected in a beam of rays parallel to the x-axis.

21. Find the several solution of $y''-y'-2y=4x^2$.

22. Solve $\frac{dx}{dt}=3x-4y,\ \frac{dy}{dt}=x-y$.

23. Use Lagrange's method to solve the equation

$$\begin{vmatrix} x & y & z \\ \alpha & \beta & \gamma \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} & -1 \end{vmatrix} = 0 \text{ where } z=z(x,y).$$

24. Find the complete integral of $x^2p^2+y^2q^2-4=0$ using Charpit's method.

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