MAY 2024 **U/ID 32508/UCM3B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Verify that the equation $\left(x^2 - 2y^2\right)dx + xydy = 0$ is a homogeneous equation.

2. Find the integrating factor of the equation $xdy + ydx + 3x^4y^4dy = 0$.

3. Solve $xy' = y$.

4. Write the condition that the two solutions $y_1(x)$ and $y_2(x)$ of $y'' + P(x)y' + Q(x)y = 0$ are linearly independent.

5. Verify that $y_1 = 1$ is the one solution of $xy'' + 3y' = 0$.

6. Eliminating the constants C_1 and C_2, find the differential equation of the family of curve $y = C_1x + C_2x^2$.

7. State Picards theorem.

8. Calculate $y_1(x)$ for the initial value problem $y' = x + y, y(0) = 1$.

9. Frame the partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 + b)$.

10. Solve $\frac{\partial^2 z}{\partial x^2} = xy$.

11. Write four special types of first order partial differential equation.

12. Find the complete integral of $pq = 1$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Determine $\left(x + \frac{2}{y}\right) dy + y\, dx = 0$ is exact and solve it.

14. Solve $xy'' - y' = 3x^2$.

15. Show that $y = C_1 e^x + C_2 e^{2x}$ is the general solution of $y'' - 3y' + 2y = 0$ on any interval, and find the particular solution for which $y(0) = -1$ and $y'(0) = 1$.

16. Find the general solution of $x^2y'' + 2xy' - 2y = 0$ where one of its solution is, $y_1 = x$.

17. Show that $f(x, y) = xy$, satisfies a Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le d$.

18. Find the general integral of the partial differential equation $y^2p - xyq = x(z - 2y)$.

19. Find the complete integral of $p^2 + q^2 = x + y$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Solve $xdy = (x^5 + x^3y^2 + y)dx$.

21. Solve $y'' - 3y' + 2y = 14\sin 2x - 18\cos 2x$.

22. Let $f(x, y)$ be a continuous function that satisfies a Lipschitz condition $|f(x, y_1) - f(x, y_2)| \le K\,|y_1 - y_2|$ on a strip defined by $a \le x \le b$ and $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip then prove that the initial value problem $y' = f(x, y), y(x_0) = y_0$, has one and only one solution $y = y(x)$ on the interval $a \le x \le b$.

23. Find the general integral of
$\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$.

24. Using Charpitz method, find the complete integral of the partial differential equation $z^2 = pqxy$.

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