MAY 2025 **U/ID 32508/UCM3B**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define linear equation.

2. Verify whether $e^y dx + \left(xe^y + 2y\right) dy = 0$ is exact.

3. Solve $y'' + y' = 0$.

4. Write the general form of second order linear differential equation.

5. Define Wronskian of two variables.

6. What is meant by trivial solution?

7. Write the Lagrange's type partial equation of first order.

8. Eliminate 'a' and 'b' from $z = ax + by + ab$.

9. Find the general solution of $y = xp + \frac{a}{p}$.

10. Write the clairaut's form.

11. Define homogeneous differential equation.

12. Define linearly independent.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Find the integrating factor of $\frac{dy}{dx} = \frac{y^2}{1-xy}$.

14. Find the solution of the initial value problem $y'' + y = 0$, $y(0) = 1$ and $y(0) = 1$.

15. Find the general solution of the system

$$\begin{cases} \frac{dx}{dt} = x + y \\ \frac{dy}{dt} = 4x - 2y \end{cases}$$

16. Find the general integral of $y^2 p - xyq = x(z - 2y)$.

17. Solve $\sqrt{p} + \sqrt{q} = 1$.

18. Eliminate arbitary function from $z = xy + f\left(x^2 + y^2\right)$.

19. Solve $xy'' - y' = 3x^2$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Solve $y'' + k^2 y = 0$.

21. Find the general solution of $y'' - y' - 2y = 4x^2$.

22. If the homogeneous system has two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ on $[a, b]$, then prove that $\begin{cases} x = c_1 x_1(t) + c_2 x_2(t) \\ y = c_1 y_1(t) + c_2 y_2(t) \end{cases}$ is also solution on $[a, b]$ for any constants c_1 and c_2.

23. Use Lagrange's method to solve the equation

$$\begin{vmatrix} x & y & z \\ \alpha & \beta & \gamma \\ \dfrac{\partial z}{\partial x} & \dfrac{\partial z}{\partial y} & -1 \end{vmatrix} = 0, \text{ where } z = z(x, y).$$

24. Find the complete integral of $\left(p^2 + q^2\right) y = qz$.
