DECEMBER 2022 **U/ID 32510/UCM4A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Find $L(\cos 3t)$.

2. Find $L(e^t + e^{-t})$.

3. Find $L(f(at))$.

4. Find the inverse Laplace transform of $\frac{2s}{s^2-9} + \frac{1}{s^3}$.

5. Find $L^{-1}\left(\frac{1}{s^2+2s+2}\right)$.

6. Find $L^{-1}(sF(s))$.

7. Write down Euler formula in $[0, 2\pi]$.

8. Define Periodic function.

9. Write down Fourier integral formula.

10. Find Fourier cosine transform of $f(x)$.

11. Find $L\left(\frac{d^2y}{dt}\right)$.

12. Solve $\frac{dy}{dt} = 2$ if $y(0) = 0$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Find $[t^2 e^{-3t}]$.

14. Find the Laplace transform of the function $f(t) = \begin{cases} t & if\ 0 < t < b \\ 2b - t & b < t < 2b \end{cases}$ with $f(t + 2b) = f(t)$.

15. Find $L^{-1}\left(\frac{s+1}{s^2 + 6s + 25}\right)$.

16. Find $L^{-1}\left(\frac{1}{s(s+1)(s+2)}\right)$.

17. Prove that the Fourier series for $f(x) = x \ -\pi < x < \pi$ is given by

$$f(x) = 2 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n}.$$

18. Find the Fourier Cosine transform of $5e^{-2x} + 2e^{-5x}$.

19. Solve $y'' + y = 2e^{t}$ if $y(0) = 1$ and $y'(0) = 2$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Verify the initial value theorem for $f(t) = e^{-2t} \sin t$.

21. Using convolution theorem find $L^{-1}\left(\frac{s^2}{(s^2 + a^2)\ (s^2 + b^2)}\right)$.

22. Determine the Fourier series of $x + x^2$ in $(-\pi, \pi)$ and deduce the sum of the series $\frac{1}{1^2} + \frac{1}{2^2} + \ldots$.

23. Solve the integral equation $\int_0^\infty f(x)\cos\lambda\,dx = \begin{cases} 1-\lambda, & \lambda \in [0,1] \\ 0 & \lambda > 1. \end{cases}$ Hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2}\,dt = \frac{\pi}{2}$.

24. Solve $\frac{dx}{dt} - \frac{dy}{dt} - 2x + 2y = 1 - 2t$; $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + x = 0$ with $x = y = 0 = \frac{dx}{dt}$ when $t = 0$.