DECEMBER 2023 **U/ID 32510/UCM4A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define Laplace Transform.
2. What are the two important theorems in laplace transform?
3. Find $L(\sin(2t+3))$.
4. What is the inverse Laplace transform method?
5. What is the formula for the convolution theorem?
6. List the properties of inverse laplace transform.
7. How do you change the interval in a Fourier series?
8. What is half range of cosine series?
9. What is the Fourier transform method?
10. Show the formula for Forward Fourier Transform.
11. What is an example of a linear ordinary differential equation?
12. What is the formula for a linear differential equation?

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. The Laplace Transform of $f(t)$ is given by, $F(S) = \frac{8}{S(S+4)}$. Find the final value of the equation using final value theorem as well as the conventional method of finding the final value.

14. Using the Laplace transform find the solution for the following equation $\partial / \partial t.y(t) = e^{(-3t)}$ with initial conditions $y(0) = 4, Dy(0) = 0$.

15. $F(S)$ is the Laplace form of a time domain function $f(t)$. Find the expiration of $f(t)$ where

$$F(s) = \frac{1}{s(s+1)}.$$

16. Find the Inverse Laplace Transformation of function, $F(S) = \frac{2}{(s+2)(s+8)}$.

17. If c is the constant in $(0 < x < \pi)$ then show that

$$c = (4c/\pi)\{\sin x + (\sin 3x/3) + \sin 5x/5) + ...\}$$

Given $f(x) = c$ in $(0, \pi)$.

18. State and prove the shifting property of Fourier Transform.

19. Find the general solution of the differential equation $xdy-(y+2x^2)dx=0$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Find the Laplace Transform of $f(t)=\sin t\begin{cases}0<t<\pi\\ t>\pi\end{cases}$

21. Compute the inverse Laplace transform of

$$Y(s)=\frac{1}{3-4s}+\frac{3-2s}{s^2+49}$$

22. Find the half range cosine series for $f(x)=x^2$ in the range $0\le x\le\pi$.

23. Prove that

$F_c[f(x)\cos ax]=1/2[F_c(s+a)+Fc(s-a)]$ where F_c denotes the Fourier Cosine transform $f(x)$.

24. Express the differential equation in Laplace transformation form $\frac{d^2x}{dt^2}+4\frac{dx}{dt}+x=1$ given $x(0^+)=0$ and $x'(0^+)=0$.

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