MAY 2022 **U/ID 32510/UCM4A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find $L(e^{-at})$.

2. Find $L[t^2 f(t)]$.

3. Evaluate $L\left[\int_0^t f(x)\,dx\right]$.

4. Find inverse Laplace transform of $\frac{1}{s^2} + \frac{1}{s+4}$.

5. Find $L^{-1}\left(\frac{1}{(s+1)^2}\right)$.

6. Find $L^{-1}\left(\frac{1}{s}F(s)\right)$.

7. Write down Fourier series of $f(x)$.

8. Find a_0 in the Fourier series of $f(x)$ in $[c, c+2\pi]$.

9. Find Fourier transform of $f(x)$.

10. Write down Parseval's identity.

11. Find $L\left(\frac{d^3y}{dt^3}\right)$.

12. Solve $\frac{d^2y}{dt^2} - 6 = 0$ if $y'(0) = 0 = y(0)$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Find $L\left(\frac{\sin 3t \cos t}{t}\right)$.

14. Find the Laplace transform of the function $f(t) = \begin{cases} t & if\ 0 < t < \pi/2 \\ \pi - t & \pi/2 < \pi \end{cases}$ and $f(\pi + t) = f(t)$.

15. Find $L^{-1}\left(\frac{3s-2}{s^2 - 4s + 20}\right)$.

16. Find $L^{-1}\left(\log \frac{s(s+1)}{s^2+1}\right)$.

17. Obtain the Fourier series of $f(x) = \pi^2 - x^2$ in $(-\pi, \pi)$.

18. Find the Fourier transform of

$$f(x) = \begin{cases} 1 & if \quad |x| < a \\ 0 & if \quad |x| > a \end{cases}.$$

19. Using Laplace transform, solve $\frac{dy}{dt} - y = 1 - 2t$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Verify final value theorem for the function $f(t) = t^2 e^{-3t}$.

21. Find $L^{-1}\left(\frac{s^2}{s^4 + 4a^4}\right)$.

22. Find the Fourier series for $f(x)$ if $f(x) = \begin{cases} -\pi & if \quad x \in (-\pi, 0) \\ x & if \quad x \in (0, \pi) \end{cases}$ and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{8}$.

23. Find Fourier transform of $e^{-a|x|}$ and hence deduce that $\int_0^{\infty} \frac{\cos xt}{a^2+t^2} dt = \frac{\pi}{2a} e^{-a|x|}$.

24. Solve $y''+2y'-3y = \sin t$ given that $y(0) = y'(0) = 0$.
