MAY 2023 **U/ID 32510/UCM4A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Find $L(t^8)$.

2. Find $L[e^{-at} f(t)]$.

3. If $L(f(t)) = F(s)$, then find $\lim_{t \to 0} f(t)$.

4. Find inverse Laplace transform of $\frac{1}{s-3} + \frac{1}{s}$.

5. Find $L^{-1}\left(\frac{1}{(s+1)^2}\right)$.

6. Find $L^{-1}[F^1(s)]$.

7. Give two examples of odd function.

8. If x is a point of discontinuity of f, then the Fourier series converges to .....

9. Write Fourier integral formula.

10. Find Fourier sine transform of $f(x)$.

11. Find $L\left(d^2y/dt^2\right)$.

12. Solve $\frac{dy}{dt} - y = 0$ if $y(0) = 0$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Find $L(f(t))$ if $f(t) = \begin{cases} \cos t & if\ 0 < t < \pi \\ \sin t & if\ t > \pi \end{cases}$.

14. Find $L\left[\frac{1-e^t}{t}\right]$.

15. Find $L^{-1}\left[\frac{1-s}{(s+1)\left(s^2+4s+13\right)}\right]$.

16. Find $L^{-1}\left(\frac{1}{\left(s^2+4\right)^2}\right)$ using convolution theorem.

17. Find the Fourier series of $f(x) = \begin{cases} 1 & if \quad x \in (0,\pi) \\ 2 & if \quad \pi < x < 2\pi \end{cases}$.

18. Find the Fourier cosine transform of e^{-x^2}.

19. Solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 5y = 5$ given that $y(0) = 0$ and $y'(0) = 2$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Verify the initial and final value theorem of the function $f(t)=1+e^{-t}(\sin t+\cos t)$.

21. Find $L^{-1}\left(\frac{1}{s(s+3)^3}\right)$.

22. Obtain the Fourier series for $f(x)=1+x+x^2$ in $(-\pi,\pi)$. Deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\ldots=\frac{\pi^2}{6}$.

23. Find the Fourier transform of $f(x)$ if $f(x)=\begin{cases}1-|x| & if\ |x|<1\\ 0 & if\ |x|>1\end{cases}$ and hence deduce that $\int_0^{\infty}\left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$.

24. Solve $\frac{dx}{dt}+y=\sin t \quad \frac{dy}{dt}+x=\cos t$ with $x=2$ and $y=0$ when $t=0$.

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