MAY 2025 **U/ID 32510/UCM4A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define Laplace Transform.

2. Prove that $L[e^{at}] = \frac{1}{s-a}$.

3. Find $L[e^{2t} + 3e^{-5t}]$.

4. Mention the Linear property of Inverse Laplace Transform.

5. State convolution theorem.

6. Find $L^{-1}\left[\frac{1}{s^2} + \frac{1}{s+4}\right]$.

7. What is meant by periodic functions?

8. Quote Half Range sine series in $(0, l)$.

9. State Fourier Integral Formula.

10. Write down the complex form of Fourier Integral.

11. Find $L[y''' - 3y'' + 3y' - y] = L\left[t^2 e^t\right]$.

12. What are the applications of boundary value problems?

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Prove that $L[\cosh at] = \dfrac{s}{s^2 - a^2}$.

14. Find the Laplace transform of the periodic function $f(t) = t$ for $0 < t < 4$ and $f(t + 4) = f(t)$.

15. Find $L^{-1}\left[\dfrac{s}{(s+2)^2 + 1}\right]$.

16. Find $L^{-1}\left[\frac{s+3}{s^2+6s+13}\right]^2$.

17. Obtain the Half Range sine series of the function $f(x) = kx(x-l)$ in $0 \le x \le l$.

18. P.T. $F_s[f(x)\cos ax] = \frac{1}{2}[F_s(s+a) + F_3(s-a)]$.

19. Solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 5y = 5$ given that $y = 0$, $\frac{dy}{dt} = 2$ when $t = 0$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Find $L\left[\frac{1-\cos t}{t}\right]$.

21. Find $L^{-1}\left[\log\frac{s(s+1)}{s^2+1}\right]$.

22. Expand $f(x) = (\pi - x)^2$ in $(-\pi, \pi)$.

23. Find the Fourier transform of $f(x)$ if $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0 & |x| > 1 \end{cases}$.

24. Solve $\frac{dx}{dt} - 2x + 3y = 0$ $\frac{dy}{dt} - y + 2x = 0$ with $x(0) = 8$, $y(0) = 3$.
