DECEMBER 2022 **U/ID 32512/UCM5A**

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define congruent.
2. Define Normal subgroup.
3. Define Kernel of ϕ.
4. Define Isomorphism.
5. Define Inner Automorphism.
6. Write the statement of Cayley theorem.
7. Define a commutative ring.
8. Define zero divisor.
9. Define Homomorphism of ring.
10. Define maximal ideal.
11. Define greatest common divisor.
12. Define relatively prime.

SECTION B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that every subgroup of cyclic group is cyclic.

14. Show that HK is a subgroup of G if and only if HK = KH.

15. If N and M are normal subgroup of G and if $N \cap M = \{e\}$ then $nm = mn$ for any $n \in N$, $m \in M$.

16. Let G be a group and ϕ be an automorphism of G and $O(a) > 0$ for all $a \in G$, then $O(\phi(a)) = O(a)$.

17. If ϕ is a homomorphism of R onto R' with kernel $I(\phi)$ then

    (a) $I(\phi)$ is a subgroup of R under addition.

    (b) If $a \in I(\phi)$ and $r \in R$ then both ar and ra are in $I(\phi)$.

18. Prove that Every Euclidean ring is a principal ideal ring.

19. If p is a prime number of the form $4n+1$ then the congruence $x^2 \equiv -1 (\text{mod } p)$.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that

    (a) Intersection of two subgroup of G is also a subgroup of G.

    (b) Union of two subgroup of G is also a subgroup of G.

21. State and prove fundamental theorem of homomorphism.

22. Prove that every finite integral domain is a field.

23. If R is a commutative ring with unit element and M is an ideal of R then prove that M is a maximal ideal of R if and only if R/M is a field.

24. Show that every integral domain can be imbedded in a field.

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