DECEMBER 2023 **U/ID 32512/UCM5A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Prove that every element in a group has a unique inverse element.

2. Define a subgroup.

3. In a group G, if $\forall\ a \in G$, $a = a^{-1}$, then prove that G is abelian.

4. Define a Normal subgroup of a group.

5. Find the generators of the cyclic group $G = \{1, -1, i, -i\}$ (Where $i^2 = -1$).

6. Find the inverse of the permutation $\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 2 & 5 & 1 \end{pmatrix}$.

7. If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ find $\alpha\beta$.

8. Give the distributive laws of rings.

9. In a ring 'R' , prove that $(-1)(-1)=1$, if R has a unit element.

10. Define an ideal in a Ring.

11. Prove that the characterics of any field is either zero or a prime number.

12. Define an Euclidean ring.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. State and prove Euler theorem.

14. If H and K are subgroups of the ablian group G, then prove that HK is a subgroup of G.

15. Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

16. Prove that the set of automorphisms of a group G is also a group.

17. Prove that every permutation is the product of its cycles.

18. If P is a prime number, then prove that J_p the ring of integers mod P is a field.

19. State and prove unique factorization theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. If H and K are finite subgroups of G of orders $O(H)$ and $O(K)$, respectively, prove that $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$.

21. State and prove Cauchy's Theorem for abelian groups.

22. State and prove Cayley Theorem.

23. Prove that R is a commutative ring with unit element and M is an ideal of R, then M is maximal ideal of R if and only if $\frac{R}{M}$ is a field.

24. Prove that every integral domain can be imbeded in a field.

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