DECEMBER 2024 **U/ID 32512/UCM5A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Prove that the intersection of two subgroups of a group is also a subgroup of G.

2. Let $a \in R^*$, $H = \{a^n / n \in Z\}$. Show that H is a subgroup of G.

3. Find the number of elements of the cyclic subgroups $< 2 >$ in $(z_{18/}, \oplus)$.

4. Define inner automorphism.

5. State Sylow's theorem for abelian groups.

6. Define Kernel of a homorphism.

7. Express the product $(1,2,3)$ $(5,6,4,1,8)$ as a disjoint cycles.

8. Compute $a^{-1}ba$ if $a = (5,7,9)$ and $b = (1,2,3)$.

9. Prove that any unit element in R cannot be a zero divisor.

10. Let R be a ring with identity 1. If U is an ideal of R and $U \in I$, Prove that U = R.

11. Define Greatest Common divisor.

12. Prove that if π is a prime element in the Euclidean ring R and π / ab where $a, b \in R$ then π divides at least one of a or b.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that G is a finite group and $a \in G$, then $o(a) | o(G)$.

14. State and prove Euler theorem.

15. If ϕ is a homomorphism of G into $\overline{G}$ with kernel k prove that k is a normal subgroup of G.

16. Prove that every permutation is the product of two cycles.

17. Prove that any finite integral domain is a field.

18. State and prove fundamental theorem of homomorphism of a ring.

19. Let R be a Euclidean ring and $a, b \in R$. Prove that if $b \neq 0$ is not a unit in R, then prove that $d(a) < d(ab)$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. State and prove Lagrange's theorem.

21. State and prove Cauchy's theorem for abelian groups.

22. State and prove Cayley's theorem.

23. Prove that if R is a commutative ring with unit element and M is an ideal of R, then M is a maximal ideal of R if and only if R/M is a field.

24. Prove that J(i) is a Euclidean ring.

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