MAY 2022 **U/ID 32512/UCM5A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define a subgroup.

2. Write a statement of Fermat theorem.

3. Define homomorphism.

4. Define isomorphic.

5. State Lagrange's theorem.

6. Define normalizer.

7. Define center of a group.

8. Define division ring.

9. Define a field.

10. Define Euclidean ring.

11. Define principal ideal ring.

12. Define relatively prime.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. If H is a non empty finite subset of a group G and H is closed under multiplication then H is a subgroup of G.

14. State and prove Euler's theorem.

15. If ϕ is a homomorphism of G into $\overline{G}$ then

    (a) $\phi(e) = \bar{e}$ is the unity element of $\vec{G}$

    (b) $\phi(x^{-1}) = [\phi(x)]^{-1} \forall\ x \in G$.

16. If R is a ring then for all $a,b,c \in R$

    (a) $a.0 = 0.a = 0$

    (b) $a(-b) = (-a)b = -(ab)$

    (c) $(-a)(-b) = ab$

    (d) $a.(b-c) = ab - bc$

    (e) $(-1).a = a.(-1) = -a$.

17. Show that the homomorphism ϕ of R into R' is an isomorphism if and only if $I(\phi) = (0)$.

18. Let R be a commutative ring with unity element whose only ideals are (0) and R itself then R is a field.

19. If U is an ideal of R, then prove that R/u is a ring.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. If H and K are finite subgroups of G of order $O(H)$ and $O(K)$ respectively then $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$.

21. If G is a group, show that $A(G)$ is the set of all automorphism of G is also a group.

22. State and prove Cayley's theorem.

23. If U is an ideal of ring R then R/U is a ring and it is a homomorphic image of R.

24. State and prove unique Factorization theorem.

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