MAY 2023 **U/ID 32512/UCM5A**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define a right coset.

2. Define cyclic group.

3. State Euler's theorem.

4. Define Quotient group.

5. Define automorphism.

6. Define a Ring.

7. State the Fundamental theorem of homomorphism in group.

8. Define integral domain.

9. Define an ideal of a ring.

10. Define Maximal ideal.

11. Define Euclidean Ring.

12. State Fermat's theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that the relation $a \equiv b \pmod{H}$ is equivalence relation.

14. State and prove Lagrange's theorem.

15. Prove N is a normal subgroup of G if and only if $gNg^{-1} = N \forall g \in G.$

16. Show that every field is an integral domain.

17. If ϕ is a homomorphism of R into R' then Prove that

    (a) $\phi(o) = o' \in R'$

    (b) $\phi(-a) = -\phi(a) \forall a \in R$

18. Prove that every Euclidean ring is a principal ideal ring.

19. Let R be an Euclidean ring suppose that a/bc but (a,b) = 1 for all a,b,c $\in R$ then a/c.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Let H be a subgroup of G then prove that any two right or left cosets of H in G are either identical or have no element in common (dis joint)

21. If G is a group then prove that A (a) is the set of all automorphism of G is also a group.

22. Prove that every finite integral domain is a field

23. If U is an ideal of ring R then prove that R/U is a Ring and is a homomorphic image of R.

24. Prove that the ideal $A=(a_o)$ is the maximal ideal of the Euclidean ring R if and only if a_o is a prime element of R.

———————