MAY 2024 **U/ID 32512/UCM5A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define the coset of a subgroup.

2. State Fermat's theorem for a group.

3. Define homomorphism of a group.

4. Explain the term quotient groups.

5. Prove that the centre of a group is a normal subgroup of G.

6. State Cayley theorem.

7. Find the orbit of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$.

8. Define division ring.

9. State Pigeonhole principle.

10. Define Boolean ring with example.

11. Define ring with unit element.

12. Prove that an Euclidean ring possesses an unit element.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that every subgroup of a cyclic group is cyclic.

14. Let N be a normal subgroup of a group G. Prove that G/N is a group under the operation defined by $NaNb = Nab$.

15. Let G be a group and φ an automorphism of G. Prove that if $a \in G$ is of order $0(a) > 0$, then $O(\varphi(a)) = o(a)$.

16. Prove that any permutation can be expressed as a product of transpositions.

17. Prove that a ring 'R' has no zero divisors iff cancellation law is valid in R.

18. If F be any field Then prove that the only ideals of F are {0} and F itself.

19. State and prove Fermat's theorem for a ring.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that HK is a subgroup of G if and only if $HK = KH$

21. State and prove Sylow's theorem for abelian groups.

22. If G is a group, H a subgroup of G and S is the set of all right cosets of H in G, then prove that there is a homomorphism θ of G into $A(S)$ and the kernal of θ is the largest normal subgroup of G which is contained in H.

23. Let R be a commutative ring with unit element whose only ideals are (0) and R itself –prove that R is a field.

24. Prove that $J[i]$ is Euclidean ring.

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