MAY 2025 **U/ID 32512/UCM5A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define group.
2. If G is a group their identity element of G is unique.
3. What is meant by homomorphism?
4. State Euler's theorem.
5. Define permutation.
6. Prove that the product of two even permutation is an even permutation.
7. What is ring theory?
8. If R is a ring then prove that $a \cdot 0 = 0 \cdot a = 0 \forall a, b \in R$.
9. Define field.
10. Define Euclidean ring.
11. What is cyclic group?
12. Define symmetric group.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. If H and K are subgroup of G and $O(H) > \sqrt{O(G)}$, $O(H) > \sqrt{O(G)}$, then prove that $H \cap K \neq (e)$.

14. Prove that N is a normal subgroup of G iff $gNg^{-1} = N$ for every $g \in G$.

15. Prove that every permutation can be uniquely expressed as a product of disjoint cycle.

16. If u, v are ideal of R. Let $U + V = \{u + v / u \in V, v \in V\}$ then prove that $u + v$ is also an ideal.

17. Prove that a Euclidean ring possesses a unit element.

18. State and prove Lagrange's theorem.

19. State and prove Fermat's theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that HK is a subgroup of G iff $HK = KH$.

21. State and prove Sylow's theorem.

22. Prove that the number of elements conjugate to 'a' in G is the index of the normalizer of 'a' in G.

23. If U is an ideal of R then prove that R/U is a ring and is a homomorphism image of R.

24. Prove that every integral domain can be embedded in a field.

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