DECEMBER 2022 **U/ID 32513/UCM5B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. If $f : I \to I$ is defined by $f(n) = n + 7 \ \forall n \in I$, then find the range of f where $I = \{1,2,3,\ldots\}$

2. Test the countability of the set of rational numbers.

3. Define least upper bound.

4. Define a sequence.

5. Find $\lim_{n \to \infty} \frac{2n}{n+3}$.

6. Give an example of oscillating sequence.

7. Find lim sup of {2, –2, 2, –2,...}

8. Define Cauchy sequence.

9. Relate conditional convergence and absolute convergence of a series.

10. When does the power series $\sum_{n=1}^{\infty} a_n x^n$ converge for all x?

11. Define a metric l^{∞}.

12. When is the function f continuous at $a \in \mathbb{R}$?

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. If $f : A \to B$ and X and Y are subsets of A, then prove that $f(X \cup Y) = f(X) \cup f(Y)$.

14. Prove that the cantor set is equivalent to [0, 1].

15. Prove that every convergent sequence is bounded.

16. If $\{s_n\}$ and $\{t_n\}$ are bounded sequence of real numbers, then prove that

$$\limsup_{n\to\infty}(s_n + t_n) \le \limsup_{n\to\infty} s_n + \limsup_{n\to\infty} t_n$$

17. Prove that every convergent sequence of real number is a cauchy sequence.

18. Prove that the series $\sum_{n=1}^{\infty} 1/n^2$ converges.

19. If $f(x) = \begin{cases} x & if \quad 0 \le x \le 1 \\ 3 - x & if \quad 1 \le x \le 2 \end{cases}$, then find $\lim_{x\to 1-} f(x)$ and $\lim_{x\to 1+} f(x)$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that [0, 1] is uncountable.

21. Prove that the sequence $\left\{(1+1/n)^n\right\}_{n=1}^{\infty}$ is convergent.

22. Prove that every cauchy sequence of real numbers is convergent.

23. State and prove the schwarz inequality.

24. If f is continuous at 'a' and if g is continuous at $f(a)$, then prove that $g \circ f$ is continuous at a.