DECEMBER 2023 **U/ID 32513/UCM5B**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. If A and B subsets of S, prove that $(A \cup B)' = A' \cap B'$.

2. Define countable sets.

3. What is the g.l.b. of $\left\{\frac{2^n - 1}{2^n}\right\}_{n=0}^{\infty}$.

4. If $S = \{s_n\}_{n=1}^{\infty} = \{2n-1\}_{n=1}^{\infty}$ find s_5 and s_9.

5. Define convergence sequence.

6. Define bounded sequence.

7. Evaluate $\lim_{n\to\infty} \frac{3n^2 - 6n}{5n^2 + 4}$.

8. State nested interval theorem.

9. If $\sum_{n=1}^{\infty} a_n$ is a convergent series, prove that $\lim_{n\to\infty} a_{n=0}$.

10. State Schwarz inequality.

11. Define Cauchy sequence for metric spaces.

12. If 'f' is continuous at 'a', and if $c \in R$, prove that cf is continuous at 'a'.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that the inverse image of the union of two sets is the union of the inverse images.

14. Prove that the set of all rational numbers is uncountable.

15. If $\{S_n\}_{n=1}^{\infty}$ is a sequence of non negative numbers and if $\lim_{n-\infty} S_n = L$, prove that $L \geq 0$.

16. Prove that the sum of two convergent sequences is convergent.

17. Prove that the Cauchy sequence of real numbers is convergent.

18. Prove that $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)$ is divergent.

19. Prove that the continuous function of a continuous function is continuous.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that countable union of countable set is countable.

21. Show that the sequence $\left\{\left(n+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.

22. Prove that any bounded sequence of real numbers has a convergent subsequence.

23. State and prove Mincoweski in equality.

24. Show that l^2 is a metric space.

———————