DECEMBER 2024 **U/ID 32513/UCM5B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. How many elements does $A = \{1, 2, \{3\}, \{4, 5\}\}$ have?

2. Define the composition of functions.

3. Define control set.

4. If $S = \{s_n\}_{i=1}^{\infty} = \{2n - 1\}_{n=1}^{\infty}$ and $N = \{n_i\}_{i=1}^{\infty} = \{i^2\}_{i=1}^{\infty}$ find n_2 and s_{n_2}.

5. If $\{s_n\}_{n=1}^{\infty}$ converges to 0. Prove that $\{(-1)^n s_n\}_{n=1}^{\infty}$ converges to 0.

6. Evaluate $\lim_{n \to \infty} \frac{2n^3 + 5n}{4n^3 + n^2}$.

7. Find the limit inferior of the sequence $A = \{1, -1, 1, -2, 1, -3, 1, -4 \ldots\}$.

8. State nested interval theorem.

9. Define alternating series.

10. State Schwarz inequality.

11. Define Cauchy sequence for the metric space.

12. If 'f' is continuous at 'a' and if $C \in R$, prove that cf is continuous at a.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that the set of all rational numbers in [0, 1] is countable.

14. State and prove least upper bound axiom.

15. If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.

16. Prove that $\lim_{n\to\infty} \frac{3n^2 - 6n}{5n^2 + 4} = \frac{3}{5}$.

17. Show that the series $\sum_{m=1}^{\infty}\left(\frac{1}{n}\right)$ is divergent.

18. If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then prove that $\sum_{n-1}^{\infty} a_n$ converges.

19. Prove that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that the countable union of the countable set is countable.

21. Prove that a nondecreasing sequence which is bounded above is convergent.

22. Prove that every Cauchy sequence is a convergent sequence.

23. If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers and if $\sum_{n=1}^{\infty} a_n$ converges, then prove that $\lim_{n\to\infty} na_n = 0$.

24. Prove that if f is continuous at a if and only if $\lim_{n\to\infty} x_n = a$ implies $\lim_{n\to\infty} f(x_n) = f(a)$.
