MAY 2022 **U/ID 32513/UCM5B**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2$, then find $f^{-1}(1)$.

2. Define a countable set.

3. If $A = \left\{ \frac{n-1}{n} \middle| n = 1,2,3,... \right\}$, then find lub of A.

4. Give an example of bounded Monotone sequence.

5. Define limit of a sequence.

6. Find one sub sequence of $\{1/n\}_{n=1}^{\infty}$.

7. Find $\lim_{n\to\infty} \frac{n^2}{n^2+3}$.

8. Define Cauchy sequence.

9. When does the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}_{a_n}$ converge?

10. List the class l^2.

11. Define discrete metric on $\mathbb{R}$.

12. Test the continuity of $f(x) = \sqrt{x}$ on $(0, \infty)$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. If $f : A \to B$ and X and Y are subsets of B, then prove that $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$.

14. Prove that the convergent sequence of real number converge to unique limit.

15. Prove that a non-decreasing sequence which is bounded above is convergent.

16. If $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ for sequence $\{s_n\}$ and $\{t_n\}$ then prove that $\lim_{n\to\infty} s_n t_n = LM$.

17. If $\lim_{n\to\infty} \sup s_n = L = \lim_{n\to\infty} \inf s_n$ for a sequence $\{s_n\}$, then prove $\{s_n\}$ converges and $\lim_{n\to\infty} s_n = L$

18. If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then prove that $\lim_{n\to\infty} a_n = 0$.

19. If f is a non-decreasing function on (a, b) that is bounded above, then prove that $\lim_{x\to b^-} f(x)$ exits.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that [0, 1] is uncountable.

21. Prove that the sequence $\{(1+1/n)^n\}_{n=1}^{\infty}$ is convergent.

22. State and prove nested – interval theorem.

23. State and prove Minkowski inequality.

24. Prove that $f : \langle M_1, \rho_1 \rangle \to \langle M_2, \rho_2 \rangle$ is continuous at $a \in M$ if and only if for $\in > 0$ there exists $\delta > 0$ such that $\rho_1(x,a) < \delta \Rightarrow \rho_2(f(x), f(a)) < \in$.

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