MAY 2023 **U/ID 32513/UCM5B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions

1. If $f : \mathbb{R} \to \mathbb{R}$ and g: $\mathbb{R} \to [0, \alpha]$ one defined by $f(x) = 1 + \sin x$ and $g(x) = x^2$, then find $g \circ f$.

2. Express χ_{AUB} in terms of χ_A and χ_B.

3. Define a countable set.

4. Relate convergence and bounded sequences.

5. Define limit of a sequence.

6. Give an example of divergence sequence.

7. Find lim sup $\{1, 2, 3, 1, 2, 3, ...\}$.

8. Find $\lim_{n \to \alpha} \left(1 + \frac{1}{n}\right)^{n+1}$.

9. Define alternating series.

10. When does the Σa_n converge absolutely?

11. Define a metric space.

12. Find $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions

13. If $f : A \to B$ and if X and Y are subsets of B, then prove that $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$.

14. Prove that convergent sequence converges to unique limit.

15. Prove that every convergent sequence is bounded.

16. If $\lim_{n \to \alpha} s_n = L$ and $\lim_{n \to \alpha} t_n = m$ for the sequence $\{s_n\}$ and $\{t_n\}$, then prove that $\lim_{n \to \alpha} (s_n + t_n) = L + M$.

17. If $\{s_n\}$ is a convergent sequence of real numbers, then prove that $\underset{n \to \alpha}{Lt} \sup\ s_n = \underset{n \to \alpha}{Lt}\ s_n$.

18. If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then prove that $\sum_{n=1}^{\infty} a_n$ an converges.

19. If f and g are continuous at a, then prove that f.g is continuous at a.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

20. Prove that $\mathbb{R}$ is uncountable

21. Prove that the sequence $\left\{2 - 1/2^{n-1}\right\}_{n=1}$ converges to 2.

22. If $\lim_{n\to\infty} \sup s_n = M$ for sequence $\{s_n\}$ then prove that

(a) $s_n < M + \in$ for all but a finite values of n.

(b) $s_n > M - \in$ for infinitely many values of n.

23. State and prove the ratio test.

24. Prove that the real - valued function f is continuous at $a \in \mathbb{R}$ if and only if (whenever) $\{x_n\}$ is sequence of real numbers converges to a then the sequence $\{f(x_n)\}$ converges to $f(a)$.