MAY 2024 **U/ID 32513/UCM5B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. If $A = \{1, 6, 7\}$ and $B = \{1, 3, 6, 7, 8\}$ find $A \cup B$ and $A \cap B$.

2. Define equivalent set.

3. State least upperbound axiom.

4. Evaluate $\lim_{n\to\infty} \frac{2n}{n+3}$.

5. Define monotone sequences.

6. Find the limit superior and limit inferior of $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$.

7. For what values of x does the series $(1+x)+(x-x^2)+(x^2-x^3)+\ldots$ converges.

8. State Minkowski's inequality.

9. Define n^{th} partial sum of the series.

10. Define absolute convergence.

11. If $|x-2|<1$, prove that $|x^2-4|<5$.

12. Define open ball.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. If $f: A \to B$ and $X \subset A$, $Y \subset A$, then prove that $f(X \cup Y) = f(X) \cup f(Y)$.

14. Prove that infinite subset of the countable set is countable.

15. If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.

16. If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers which converges to L, prove that $\{s_n^2\}_{n=1}^{\infty}$ converges to L^2.

17. State and prove nested interval theorem.

18. State and prove Schwarz inequality.

19. Prove that every function from R_d into a metric space is continuous on R_d.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that the set $[0, 1] = \{x \mid 0 \le x \le 1\}$ is uncountable.

21. Prove that a nondecreasing sequence which is bounded above is convergent.

22. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers, then prove that

$\limsup_{n\to\infty}(s_n + t_n) \le \limsup_{n\to\infty} s_n + \limsup_{n\to\infty} t_n$ and

$\liminf_{n\to\infty}(s_n + t_n) \ge \liminf_{n\to\infty} s_n + \liminf_{n\to\infty} t_n$.

23. State and prove ratio test.

24. Prove that the real valued function f is continuous at $a \in R'$ if and only if whenever $\{x_n\}_{n=1}^{\infty}$ is a sequence of real number converging to a, then the sequence $\{f(x_n)\}_{n=1}^{\infty}$ converges to $f(a)$.

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