MAY 2025 **U/ID 32513/UCM5B**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. How can you write if b is an element of the set A?
2. Define equal set.
3. What is called an upper bound?
4. Define sequence.
5. What is called divergent sequence?
6. What is meant by monotone increasing sequence?
7. Quote the sum of divergent sequence.
8. Define Greatest Lower Bound.
9. What is called an alternating series? Give an example.
10. Define summability of series.
11. What is called right hand limit of f at a?
12. Define reformulation of function f is continuous at $a \in R'$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. If A, B are subsets of $\overline{S}$ then prove that $(A \cup B)' = A' \cap B'$.

14. If $F : A \to B$ and $X \subset A$, $Y \subset A$ then prove that $F(X \cup Y) = f(x) \cup F(y)$.

15. Prove that the sequence $\left\{\frac{n^2}{n+5}\right\}_{n=1}^{\alpha}$ has a limit or not.

16. Suppose $\{S_n\}_{n=1}^{\alpha}$ converges to 0 prove that $\{(-1)^n S_n\}_{n=1}^{\alpha}$ converges to 0.

17. Prove that $\lim_{n\to\alpha} \frac{2n^3 + 5n}{4n^3 + n^2} = \frac{1}{2}$.

18. If $(a_n)_{n=1}^{\alpha}$ is a non increasing sequence of positive numbers and if $\sum_{n=1}^{\alpha} a_n$ converges then prove that $\lim_{n\to\alpha} na_n = 0$.

19. If $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} g(x) = M$ then $f(x) + g(x)$ has a limit as $x \to a$ and prove that $\lim_{x\to a}[f(x) + g(x)] = L + M$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Show that the countable union of countable set is countable.

21. Let $S = \{S_n\}_{n=1}^{\alpha}$ be a sequence of real numbers. Prove that S has a monotone subsequence.

22. State and prove cauchy sequence of first proof of the theorem.

23. State and prove Schwarz Inequality.

24. In the real valued functions f and g are continuous at $a \in R'$ then prove that $f+g$, $f-g$ and fg. If $g(a) \neq 0$ then f/g is also continuous at a.

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