DECEMBER 2022 **U/ID 32514/UCM5C**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. List all the subsets of the set $A = \{a, b, c\}$.

2. Define greatest common divisor.

3. State the principle of mathematical induction.

4. Prove that $a \cdot (a + b) = a$ for all $a, b \in B$.

5. Define dual in Boolean algebra.

6. Define disjunctive normal form.

7. Write the AND gate symbol to represent the Boolean expression $x_1, x_2, \ldots x_n$.

8. Draw the circuit representing the Boolean function $x \vee y$.

9. Define sequence.

10. Define generating function.

11. Define simple graph.

12. Define Euler circuit.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. If a,b are positive integers the prove that $\gcd(ka,kb) = k\gcd(a,b)$ for any positive integer k.

14. Use mathematical induction to prove $1^2 + 2^2 + 3^3 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$, where $n \geq 1$.

15. Construct the truth table for the Boolean expression of $f(x,y,z) = (x \cdot y) + z^1$.

16. Draw a circuit for the Boolean function $l + m$, $l + n$ where $l = x \cdot y$, $m = z + t$, $n = z \cdot t$.

17. Solve the recurrence relation $a_n = 8_{an-1} - 16_{an-2}$ with initial conditions $a_2 = 16$ and $a_3 = 80$.

18. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$ if $a_n = 7.2 - n + 2$.

19. Prove that the sum of the degrees of all vertices of a graph is an even integer.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. If A, B, and C are three sets then prove that $A \cap (B \mid C) = (A \cap B) \setminus (A \cap C)$.

21. Express the Boolean function $f = [(xy') + z'] \cdot [x' \cdot z]'$ in CNF.

22. The Fibonacci sequence $\{f_n\}_n^{\infty} = 0$ is defined by $f_n = f_{n-1} + f_{n-2}$; $n \geq 2$ with the conditions $f_0 = 0$ and $f_1 = 1$. Find an explicit formula.

23. Use generating function to solve the recurrence relation $a_n = 3a_{n-1} + 2$, $n \geq 1$ with initial condition $a_0 = 1$.

24. Prove that a connected graph 'G' has an Eulerian trail if and only if it has atmost two odd vertices.

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