DECEMBER 2023 **U/ID 32514/UCM5C**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define a Set.

2. If $U = \{a,b,c,d,e,f\}$, $A = \{a,b,c\}$, $B = \{c,d,e,f\}$, $C = \{c,d,e\}$, find $(A \cap B) \cup (A \cap C)$.

3. What is the proof of divisibility of integers?

4. Define identity law.

5. What is conjunctive normal form?

6. State the commutative properties of Boolean algebra.

7. What is a combinatorial logic circuit?

8. Show the truth table for AND Gate.

9. How do you solve a recurrence relation with a constant?

10. What is the sum of two discrete numeric functions?

11. Define a Walk.

12. What is a cycle?

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that if x is even, then $x^2 + 2x + 4$ is divisible by 4.

14. Give a brief note on working rule of Mathematical induction.

15. Prove that $x.y + (x' + y') = 1$ and $x.y + (x' + y') = 0$.

16. Obtain Disjunctive Normal Form of $Q \vee (P \wedge R) \vee \neg ((P \vee R) \wedge Q)$.

17. Find the values of the Boolean function represented by $F(x, y, z) = x \cdot y + \bar{z}$.

18. Solve the recurrence relation $a_n = 7a_{n-1} + 10a_{n-2}$ with $a_0 = 2$ and $a_1 = 3$.

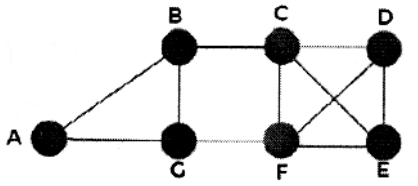
19. Find out which sequence of the vertices determines walks. The sequence is described below:

(a) A, B, G, F, C, D

(b) B, G, F, C, B, G, A

(c) C, E, F, C

(d) C, E, F, C, E



PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. By using mathematical induction prove that the given equation is true for all positive integers.

$1 \times 2 + 3 \times 4 + 5 \times 6 .... + (2n-1) \times 2n = n(n+1)$
$(4n-1)/3$.

21. Determine the PDNF with the help of truth table for the expression $(X \wedge Y) \vee (\neg X \wedge Z) \vee (Y \wedge Z)$.

22. Design a logic circuit that has three inputs, A, B and C and whose output will be HIGH only when a majority of the inputs are HIGH.

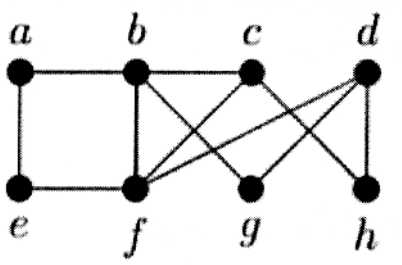
23. Consider the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$.

(a) Find the general solution to the recurrence relation.

(b) Find the solution when $a_0 = 1$ and $a_1 = 2$.

(c) Find the solution when $a_0 = 1$ and $a_1 = 8$.

24. In the graph



(a) Find a path of length 3.

(b) Find a cycle of length 3.

(c) Find a walk of length 3 that is neither a path nor a cycle. Explain why your answer is correct.
