MAY 2022 **U/ID 32514/UCM5C**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. If $U = \{1,2,3,4,5\}$ $A = \{1,5\}$ and $B = \{1,2,3,4\}$ find $\overline{A} \cup \overline{B}$.

2. State principle of mathematical induction.

3. Define divisibility.

4. Define Boolean algebra.

5. Prove that $a + a.b = a$ for all $a,b \in B$.

6. Define conjunctive normal form.

7. Write the OR gate symbol to represent the Boolean expression $x_1, x_2, ...x_n$.

8. Draw the circuit representing the Boolean function $x \wedge (y \vee z)$.

9. Define recurrence relation.

10. Find the generating function for $(1+x)^{-n}$ where 'n' is a positive integer.

11. Define complete graph.

12. Define connected graph.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. State and prove DeMorgan's laws.

14. Use mathematical induction to prove the inequality $n < 2^n$ for all positive integers n.

15. In any Boolean algebra, show that $(a + b')(b + c')(c + a') = (a'+b)(b'+c)(c'+a)$.

16. Draw a circuit for the Boolean function $l + m$, $l.m$ where $l = x.y$, $m = z + t$.

17. Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 6$.

18. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n - a$ if $a_n = -n + 2$.

19. Prove that the number of odd degree vertices is always even.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. If 'A' and 'B' are two sets, prove that $(A \mid B) \cup (B \mid A) = (A \cup B) \mid (A \cap B)$.

21. Express the Boolean function $f = \left[(x'y + (xyz') + (xy'z) + (x'y'z't) + t'\right]$ in DNF.

22. The Fibonacci sequence $\{f_n\}_{n=0}$ is defined by $f_n = f_{n-1} + f_{n-2}$; $n \geq 2$ with the conditions $f_0 = 0$ and $f_1 = 1$. Find an Explicit formula.

23. Use generating functions to solve the recurrence relation $a_{n+2} - 2a_{n+1} + a_n = 2^n$ with the initial conditions $a_0 = 2$ and $a_1 = 1$.

24. Prove that a simple graph with n-vertices and K components can have atmost $\frac{(n-K)(n-K+1)}{2}$ edges.

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