MAY 2023 **U/ID32514/UCM5C**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define Super set.

2. If $A=(a,b)\,and\,B=(\alpha,\beta)$, Find $A\times B$.

3. Define divisibility.

4. Define Boolean algebra.

5. Prove that a.(a+b)=a for all $a,b\in B$.

6. Define Disjunctive Normal form.

7. Write the AND gate symbol to represent the Boolean expression $x_1, x_2 \ldots .x_n$.

8. Draw the circuit for the Boolean function $x \wedge (y \vee z)$.

9. Define Recurrence relation.

10. Define generating function.

11. Define Multigraph.

12. Define Eulerian path.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions,

13. If A,B and C are sets, then prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

14. Use Mathematical induction to prove that $n^3 - n$ which is divisible by '3', n is a positive integer.

15. Prove that $a + 1 = 1$ and $a.0 = 0$ for all $a \in B$.

16. Draw a circuit for the Boolean function

$$x.[(y + t') + (z'.(x + t + z'))].y$$

17. Solve the recurrence relation

$a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$.

18. Show that the sequence (a_n) is a solution of the recurrence relation

$$a_n = a_{n-1} + 2a_{n-2} + 2n - 9$$

if $a_n = 3(-1)^n + 2^n - n + 2$. Prove that

19. Prove that if a graph G has exactly two vertices of odd degree, then there is a path joining these two vertices.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove : Let 'a' be any integer and 'b' be a positive integer, Then there exist unique integers 'q' and $'r'$ such that $a = qb + r$, where $0 \le r < b$.

21. Verify Demergan's Laws in Boolean algebra.

22. Use generating function to solve the recurrence relation $a_n = 3a_{n-1} + 2,\ n \ge 1$ with initial condition $a_0 = 1$.

23. Prove that a simple graph with n-vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

24. Prove that a connected graph G is Eulerian if and only if every vertex of G has even degree.

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