DECEMBER 2022 **U/ID 32516/UCM6A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Give an example of subspace of $\mathbb{R}^3$ over $\mathbb{R}$.

2. Define basis of a vector space.

3. Prove that the vectors (1,2,3), (0, 1, 2) and (0,0,1) are linearly independent in $\mathbb{Q}^3$ over $\mathbb{Q}$.

4. If $\dim_F V = 3$ and $\dim_F W = 4$, then find $\dim(Hom(V,W))$.

5. If W is a subspace of V, then find $A(A(W))$.

6. Prove that $(0,v) = 0 \forall v \in V$ where V is an inner product space.

7. Give an example of an inner product space.

8. If W is a subspace of an inner product space V, then prove that $W^{\perp}$ is a subspace of V.

9. Define an Algebra.

10. When do you say that $T \in A(V)$ is onto?

11. Define similar linear transformation.

12. Define matrix of $T \in A(V)$ in the basis $v_1, \ldots v_n$

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that $L(S \cup T) = L(S) + L(T)$.

14. If W is a subspace of V, then prove that V/W is a vector space.

15. If V is finite dimensional and $v \neq 0 \in V$, then prove that there is a $f \in \hat{V}$ such that $f(v) \neq 0$.

16. Prove that every orthonormal set is linearly independent is an inner product space.

17. If V is finite dimensional over F_2 then prove that $T \in A(V)$ is singular if and only if there is a $v \neq 0$ s.t. $T(v) = 0$.

18. If λ is a characteristic root of $T \in A(V)$, then prove that λ is a root of minimal polynomial.

19. If $W \subset V$ is invariant under $T \in A(V)$, then prove that T induces a linear transformation T on V/W, defined by $\overline{T}(v + W) = T(v) + W$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. If A and B are finite dimensional subspaces of V then prove that

$$\dim(A + B) = \dim A + \dim B - \dim(A \cap B)$$

21. If V and W are vector spaces over F of dimensions m and n respectively, then prove that Hom(V,W) is of dimension m.n.

22. Prove that every finite dimension inner product space has an orthonormal set as basis.

23. If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of minimal polynomial is not zero.

24. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is basis in which matrix of T is triangular.

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