DECEMBER 2023 **U/ID 32516/UCM6A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. If V is a vector space over F, then prove that $OV = O$ for $v \in V$.

2. Define basis of a vector space.

3. Prove that (1, 1, 0, 0), (0, 1, –1, 0) and (0, 0, 0, 3) in Q^4 are linearly independent.

4. If W is a subspace of V, $\dim V = 8$ and $\dim W = 3$, then find $\dim(A(W))$.

5. Define annihilator of a subspace W of V.

6. Give an example of an inner product space.

7. Find W^T if W is a subspace of an inner product space V.

8. Define an orthonormal set in an inner product space.

9. Define singular linear transformation.

10. Find the rank of linear transformation T on V.

11. Define similar linear transformation.

12. When do you see that the subspace W is invariant under $T \in A(V)$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that $L(S)$ is subspace of V.

14. If V is finite dimensional and $v \neq 0 \in V$, then prove that there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

15. Prove that $A(A(W)) = W$.

16. Prove that every finite dimensional inner product space has an orthonormal set as basis.

17. If V is finite dimensional over F, then prove that $T \in A(V)$ is singular is and only if there exits $v \neq 0 \in V$ such that $T(v) = 0$.

18. Prove that $\lambda \in F$ of characteristic root of $T \in A(V)$ if and only if $T(0) = \lambda 0$ for $v \neq 0 \in V$.

19. If V is the vector space of polynomials of degree 4 or less over F, then find the matrix of a differential operator in the basis $\{1, x, x^2, x^3, x^4\}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. If A and B are finite dimensional subspace of V over F, then prove that

$\dim AB = \dim A + \dim B - \dim A \cap B$.

21. If V and W are vector space of dimensions m and n respectively over F, then prove that $Hom(V,W)$ is of dimension mn.

22. If W is a subspace of finite dimensional inner product space V, then prove that $V = W \oplus W^{\perp}$.

23. If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero.

24. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis in which matrix of T is triangular.

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