MAY 2022 **U/ID 32516/UCM6A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define basis of a vector space.

2. If V is finite dimensional and W is to subspace, then find $\dim\left(\frac{V}{W}\right)$.

3. Prove that the vectors (1, 1, 1), (1, 0, 1) and (0, 1, 1) are linearly independent in $\mathbb{R}^3$.

4. Find the dual space of a vector space V.

5. If W is a subspace of a finite dimensional space V, then find $\dim(A(W))$.

6. Give an example of an inner product space.

7. Prove that $(u, \alpha v + \beta w) = \overline{\alpha}(u,v) + \overline{\beta}(u,w)$ for $u,v,w \in V$ $\alpha,\beta \in F$ and V is an inner product space.

8. Define an orthonormal set of an inner product space.

9. Find the rank of $T \in A(V)$.

10. Define characteristic vector of $T \in A(V)$.

11. Find the matrix of $T \in A(V)$ in the basis $v_1,..,v_n$.

12. When will the matrix of $T \in A(V)$ triangular?

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. If S and T are subsets of a vector space V, then prove that $L(S \cup T) = L(S) + L(T)$.

14. If V is finite dimensional and $v \neq 0 \in V$, then prove that there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

15. Prove that $A(A(W)) = W$.

16. If $u,v \in V$, then prove that $|(u,v)| \leq \|u\|.\|v\|$.

17. If V is finite – dimensional over F, then prove that $T \in A(V)$ is singular if and only if there exists $v \neq 0 \in V$ such that $T(v) = 0$.

18. If $\lambda \in F$ is characteristic root of $T \in A(V)$, then prove that λ is a root of the minimal polynomial of T.

19. If V is the vector space of polynomials over F of degree 3 or less and $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{pmatrix}$ is the matrix of $T \in A(V)$ in the basis $1, x, x^2, x^3$, then find the matrix of T in $1, 1+x, 1+x^2, 1+x^3$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. If A and B are finite – dimensional subspaces of a vector space V, then prove that

$$\dim(A+B) = \dim A + \dim B - \dim(A \cap B).$$

21. If V and W are of dimensions m and n respectively, over F, then prove that $Hom(V,W)$ is of dimension mn over F.

22. If V is a finite-dimensional inner product space, then prove that V has an orthonormal set as a basis.

23. If V is finite – dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial is not zero.

24. If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfied a polynomial of degree n over F.
