MAY 2024 **U/ID 32516/UCM6A**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define Vector Space.

2. Show that the vectors $(1, 4, -2), (-2, 1, 3)$ and $(-4, 11, 5)$ are linearly dependent.

3. Define linear span.

4. If $\dim_F V = 5$, find $\dim_F Hom(V, F)$

5. Define annihilator.

6. Show that $\|\alpha u\| = |\alpha| \, \|u\|$, $\alpha \in F$, $u \in V$.

7. Define Orthonormal set.

8. Find the norm of vector $(1, 1, 1)$.

9. Define algebra.

10. If V is finite dimensional over F and if $T \in A(V)$ is right invertible, prove that it is invertible.

11. Compute $\begin{pmatrix} 1 & 6 \\ -6 & 1 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix}$

12. Define Similar linear Transformation.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Prove that $L(S)$ is a subspace of V.

14. Prove that any two finite – dimensional vector space over F of the same dimension are isomorphic.

15. Prove that $A(A(W)) = W$.

16. If a, b, c are real numbers such that $a > 0$ and $a\lambda^2 + 2b\lambda + c \geq 0$ for all real numbers λ, prove that $b^2 \leq ac$.

17. If V is a finite–dimensional inner product space and W is a subspace of V, Prove that $(W^{\perp})^{\perp} = W$.

18. Prove that the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $\nu \neq 0$ in V, $\nu T = \lambda \nu$

19. Prove that $m(S+T) = m(S) + m(T)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. If V is the internal direct sum of $U_1, \ldots U_n$ prove that V is isomorphic to the external direct sum of $U_1, \ldots U_n$.

21. If V is finite dimensional and W is a subspace of V, prove that $\hat{W}$ is isomorphic to $\hat{V} / A(W)$ and $\dim A(W) = \dim V - \dim W$.

22. State and prove Schwarz inequality.

23. Prove that if V is finite dimensional over F, then $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero.

24. If $T \in A(V)$ has all its characteristic roots in F, prove that there is a basis of V in which the matrix of T is triangular.

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