DECEMBER 2022 **U/ID 32517/UCM6B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions

1. Prove that every subset of $\mathbb{R}_d$ is open.

2. Define limit point of a set.

3. When do you say that a real-valued function is not continuous at a?

4. Define E-dense set.

5. State finite intersection property.

6. List two complete metric spaces.

7. Compare $L[f : z]$ and $L[f : \sigma]$ if z is a refinement of σ on [a, b].

8. Give an example of a function which is note Riemann integrable on [a, b].

9. When does the continuous real-valued function f have maximum at $c \in (a,b)$?

10. State first fundamental theorem on calculus.

11. Write down Taylor series for e^x on $(-\infty,\infty)$.

12. Define uniform convergence.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions

13. Prove that every open ball in a metric space is open.

14. If A and B are sets of first category, then prove A$\cup$B is also of first category.

15. Prove that ρ^2 is complete.

16. If the metric space M has Heine-Borel propertyes, then prove that M is compact.

17. If $f \in R[a,b]$, then prove that $\left|\int_a^b f\right| \leq \int_a^b |f|$

18. State and prove Rolle's theorem.

19. Prove that {fn} converges uniformity where $f_n(x) = \frac{x}{1+nx}, 0 \leq x < \infty \forall n$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

20. State and prove the necessary and sufficient condition for a subset A of IR is connected.

21. State and prove Picard fixed point theorem.

22. Prove chat a bounded function $f \in R[a,b]$ if and only if, each $\in > 0$, there exists a subdivision σ of [a, b] such that $U(f,\sigma) < L(f,\sigma) + t$.

23. State and prove the second fundamental theorem on calculus.

24. Derive Taylors formula with integral form of remainder.

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