DECEMBER 2023 **U/ID 32517/UCM6B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Give an example of nowhere dense set in $\mathbb{R}$.

2. Define limit point of αx^t.

3. When do you say that A is a connected set in a metric space M?

4. Find the diameter of set $\{(x, y) | x^2 + y^2 \leq 1\}$ on $\mathbb{R}^2$.

5. List any two complete metric space.

6. Define contraction.

7. Validate $\int_a^b f > \int_{\underline{a}}^{b}$.

8. State necessary and sufficient condition for the existence of Riemann integral.

9. Find $(f / g)'(c)$ if $f'(c)$ and $g'(c) \neq 0$ to exist.

10. State Rolle's theorem.

11. Write down Lagrange form of remainder in Taylor's formula.

12. State Cauchy criterion for uniform convergence.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. If a subset E of a metric space is closed, then prove that $E = \overline{E}$.

14. If $f : \mathbb{R} \to \mathbb{R}$ and D is set of points in $\mathbb{R}$ at which f is not continuous then prove that D is F_0 set.

15. Prove that every closed subspace of complete metric space is complete.

16. If f and $g \in \mathcal{R}[a, b]$, then prove that $f + ge \mathcal{R}[a,b]$.

17. If f has a derivative at $c \in \mathbb{R}$, then prove that f is continuous at c.

18. If f has a derivative at each point of $[a, b]$, then prove that f' takes every value between $f'(a)$ and $f'(b)$.

19. Prove that $\{f_n\}$ converges pointwise on [0,1] where $f_n(x) = x^n$ on $[0,1]$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Prove that a subset A of $\mathbb{R}$ is connected if and only if whenever $a \in A$, $b \in A$ with $a < b$ then $c \in A$ for any c such that $a < c < b$.

21. Let A be a subset of metrispace (M_1, ρ_1). If $f : (A, \rho_1) \to (M_2, \rho_2)$ is uniformly continuous where μ_2 is complete, then prove that f can be extended to a uniformly continuous function $F : M_1 \to M_2$.

22. Prove that a bounded function f on $[a,b] \in R$ [a, b] if and only if f is continuous at almost every point in [a, b].

23. State and prove the first fundamental theorem on calculus.

24. State and prove the Dini's theorem.
