DECEMBER 2024 **U/ID 32517/UCM6B**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define closure subset of a metric space M .

2. Prove that the union of infinite number closed sets need not be closed.

3. State Barie's category theorem.

4. Define totally bounded set.

5. Define complete metric space.

6. When will you say the subset of a metric space has finite intersection property?

7. Define upper sum.

8. Using Riemann integrable function evaluate $\int_1^3 [2x - 3]dx$.

9. State Chain rule.

10. State first fundamental theorem of calculus.

11. State Taylor's formula with Lagrangian form of remainder.

12. Define point-wise convergence.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Prove that if G_1 and G_2 are open subsets of the metric space M, then $G_1 \cap G_2$ is also open.

14. Let $\langle M, P \rangle$ be a metric space and let A be a proper subset of M. Prove that the subset G_A of A is an open subset of $\langle A, P \rangle$ if and only if there exists an open subset G_M of $\langle M, P \rangle$ such that $G_A = A \cap G_M$.

15. Prove that closed subset of a complete metric space is also complete.

16. Let $\langle M_1, P_1 \rangle$ be a compact metric space. If f is a continuous function from M_1 into a metric space $\langle M_2, P_2 \rangle$, prove that f is uniformly continuous on M_1.

17. Prove that every countable subset of R' has measure zero.

18. Prove that if f has a derivative at every point of $[a, b]$ then f' takes on every value between $f'(a)$ and $f'(b)$.

19. If $M \in R$ is not a non-negative integer then $(Hx)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \cdots$ $\frac{m(m-1)\ldots(m-n+1)}{n!}x^{n+\cdots}$ provided that $|\alpha| < 1$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that the subset A of R' is connected if and only if whenever $a \in A$, $b \in A$ with $a \subset b$, then $c \in A$ for any c such that $a < c < b$.

21. Prove that M is a compact metric space if and only if it has the Heien – Borel property.

22. Let f be founded function on the closed bounded interval $[a,b]$. Prove that $f \in R[a,b]$ if and only if, for each $\varepsilon > 0$, there exists a subdivision σ of $[a,b]$ such that $U[f;\sigma] < L[f;\sigma] + \varepsilon$.

23. State and prove law of the mean.

24. State and prove Dini's theorem.
