MAY 2022 **U/ID 32517/UCM6B**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Find $B(1, 1/2)$ in absolute metric on $\mathbb{R}$.

2. Define a limit point of a set *E*.

3. Verify that (1/2, 1] is open in [0, 1].

4. Find the diameter of [–2, 3] in absolute metric on $\mathbb{R}$.

5. Give two examples of complete metric spaces.

6. State Hiene – Borel property.

7. Define a statement holds almost every where.

8. If τ is fines than σ, then compare $L[f, \tau]$ and $L[f, \sigma]$.

9. Verify that *f* has a derivative at *c* if f is continuous at *c*.

10. Find the derivativate of *f.g* if *f* and *g* have derivatives at *c*.

11. Write down the integral form of the remainder in Taylor's formula for f on $[a, a+h]$.

12. Define uniform convergence of the sequence $\{f_n\}$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Prove that every open subset of $\mathbb{R}$ can be written as a countable union of open intervals which are mutualy disjoint.

14. If $\{f_n\}$ is a sequence of closed bounded subsets of complete metric space M, such that $F_1 \supset F_2 \supset F_3 \ldots$ and $\underset{n\to\infty}{diam}\, F_n = 0$, then prove that $\bigcap_{n=1}^{\infty} F_n$ contains precisely one point.

15. Prove that a continuous function on a compact metric space is uniformly continuous.

16. If f is bounded function on $[a,b]$, σ and τ are subdivisions of $[a,b]$, then prove that $U[f;\sigma] \geq L[f;\tau]$.

17. If $f, g \in \Re[a,b]$ and $f(x) \leq g(x)$ almost everywhere, then prove that $\int_a^b f \leq \int_a^b g$.

18. State and prove Rolle's theorem.

19. Prove that the sequence of functions $\{f_n\}$ where $f_n(x) = \frac{x}{1+nx} \forall\, x \in [0, \infty)$ converges uniformly.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Prove that $f : (M, \rho_1) \to (M_2, \rho_2)$ is continuous if and only if $f^{-1}(G)$ is open in M_1 wherever G is open in M_2.

21. State and prove Picard fixed – point theorem.

22. Prove that a bounded function f on $[a,b]$ is continuous at almost $\forall$ every point in $[a,b]$ if and only if $f \in \Re[a,b]$.

23. State and prove that law of means.

24. State and prove Dini's theorem.

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