MAY 2024 **U/ID 32517/UCM6B**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Define open set.

2. What is no where dense subset?

3. Define connected set.

4. Define ε-dense subset.

5. State Heine Borel property.

6. Prove that the continuous real valued function on a closed bounded interval in R' is bounded.

7. When do you say that the function is Riemann integrable?

8. If $f, g \in \mathcal{R}[a, b]$ and if $f(\alpha) \leq g(x)$ almost every where $(a \leq x \leq b)$, prove that $\int_a^b f \leq \int_a^b g$.

9. State law of mean.

10. Find a suitable point C of Rolle's theorem for $f(x)=(x-a)(b-x),\ (a \le x \le b)$.

11. State the Taylor's formula with Lagrangian form of remainder.

12. Define uniform convergence of a sequence of function.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. If $\mathscr{F}$ is any family of closed subsets of metric space M, prove that $\bigcap_{F \in \mathscr{F}} F$ is closed.

14. Prove that if A_1 and A_2 are connected subsets of a metric space M and if $A_1 \cap A_2 \neq \varphi$, then $A_1 \cup A_2$ is also connected.

15. Prove that closed subset of a complete metric space is also complete.

16. Prove that, if the subset A of the metric space $\langle M, P \rangle$ is totally bounded then A is bounded.

17. If $f \in \mathscr{R}[a, b]$ and λ is any real number, prove that $\lambda f \in \mathscr{R}[a, b]$ and $\int_a^b \lambda f = \lambda \int_a^b f$.

18. State and prove Rolle's theorem.

19. State and prove Dini's theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. State and prove Baire category theorem.

21. Prove that l^2 is complete.

22. If $f \in \mathcal{R}[a, b]$, $g \in \mathcal{R}[a, b]$, prove that $f + g \in \mathcal{R}[a, b]$ and $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

23. State and prove chain rule.

24. Derive Taylor's formula with the integral form of remainder.

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