DECEMBER 2023 **U/ID 32518/UCM6C**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Write down Laplace's equation.

2. Evaluate $\underset{z\to\infty}{Lt}\dfrac{2z+i}{z+1}$.

3. Define conformal mapping.

4. Solve $w=\dfrac{az+b}{cz+d}$ for $z(ad-bc\neq 0)$.

5. Define contour integral.

6. Find the value of $\int_c \dfrac{dz}{z}$ where C is the semicircle.

7. Define convergence of sequence.

8. Define a zero of order m at z_0.

9. Define isolated singular points.

10. If $z=e^{i\theta}$, then find the value of $\frac{1}{2}\left(z+\frac{1}{z}\right)$.

11. Evaluate $\int_c \frac{e^z}{z}dz$ where C is the circle $|z|=1$.

12. Expand $\cos z$ into a Taylor's series about the point of $z=\frac{\pi}{2}$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Show that the function $u=\frac{1}{2}\log\left(x^2+y^2\right)$ is harmonic.

14. Obtain C-R equation in polar coordinates.

15. State and prove fundamental theorem of algebra.

16. Find Laurent series of $f(x)=\frac{1}{z^2(1-z)}$ $(0<|z|<1)$.

17. State and prove argument principle.

18. Discuss the transformation $w=\frac{1}{z}$.

19. Prove that the all bilinear transformations is group under composition.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Derive a necessary condition for $f(x)$ to be analytic.

21. Discuss the transformation $w = \sin z$.

22. Derive Canchy's integral formula.

23. State and prove Taylor's theorem.

24. Evaluate $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}$ $a > |b| > 0$.

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