DECEMBER 2024 **U/ID 32518/UCM6C**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Express $f(z) = \frac{1}{z}$ in the form $u(x, y) + iv(xy)$.

2. Define limit of a function.

3. Find the derivative of $f(z) = \frac{(2 + z^3)^4}{z - 1}; z \neq 1$.

4. Find the critical point of $w = \frac{1}{2}\left(z + \frac{1}{z}\right)$.

5. Define fixed point.

6. Define conformal mapping.

7. Expand e^z in a Taylor series about $z = 0$.

8. Define connected region.

9. State Cauchy's integral formula.

10. Find the zeros of $f(z) = z^2 \sin z$.

11. Determine the pole of $\tan hz$.

12. Find the residue of $\dfrac{1 + e^2}{\cos z + \sin z}$ at $z = 0$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Show that the function $f(z) = \bar{z}$ is nowhere differentiable.

14. If $f(z) = u(x, y) + iv(x, y)$ is an analytic function and $u(x, y) = \dfrac{\sin 2x}{\cosh 2y + \cos 2x}$, find $f(z)$.

15. Find the image of the circles $|z - 2i| = 2$ under the transformation $w = 1/z$.

16. Show that the transformation $w = \sin z$ maps the region $0 \le x \le \pi/2, y \ge 0$ onto the first quadrant of the w – plane.

17. Evaluate $\int_c |z|\bar{z}\, dz$ where C is the closed curve consisting of the upper semicircle $|z| = 1$ and the segment $-1 \le x \le 1$.

18. Find the Laurent's series expansion for $H(z) = \dfrac{1}{(z+1)(z+3)}$ in the region $0 < |z+1| < 2$.

19. Using Residue Theorem evaluate $\int_c \dfrac{z+1}{z^2+2z+4}\, dz$ where C is circle $|z+1+i| = 2$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Derive $C - R$ equations in polar form.

21. Discuss the transformation $w = \sin z$.

22. Evaluate $\int_c \dfrac{z+4}{z^2+2z+3}\, dz$ where C is $|z+1-i| = 2$.

23. State and prove Laurent's theorem.

24. Evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)} dx, \left(a > b > 0\right)$
