MAY 2022 **U/ID 32518/UCM6C**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Define Analytic Function.

2. State the sufficient condition for $f(z)$ to be analytic.

3. Determine whether $f(z)=(x^2-y^2-2xy)+i(x^2-y^2+2xy)$ is an analytic function.

4. Define conformal mapping and isogonal mapping.

5. Find the invariant points of the transformation $w=\frac{1+z}{1-z}$.

6. Define Simply Connected and Multiply Connected Regions.

7. State Cauchy-Goursat's Theorem.

8. State Laurent's theorem.

9. Define essential singularity with an example.

10. State Rouche's theorem.

11. Evaluate $\int_C \frac{z}{(z-2)^2} dz$, where C is the circle $|z| = 3$.

12. State and prove Liouville's Theorem.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Show that an analytic function with constant modulus is constant.

14. If $f(z)$ is analytic function of z and $f(z) \neq 0$ prove that $\nabla^2(\log|f(z)|) = 0$ and $\nabla^2(ampf(z)) = 0$.

15. Find an analytic function whose imaginary part is $e^{-x}(x\cos y + y\sin y)$.

16. Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$.

17. State and prove Cauchy's integral formula for derivative.

18. Expand $\frac{1}{(z+1)(z+3)}$ in Laurent's series valid in the region $1 < |z| < 3$.

19. State and prove the Argument Principle.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. Derive the Cauchy-Riemann equations in polar coordinates.

21. Prove that the function defined by

$$f(z)=\begin{cases}\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, & z\neq 0\\ 0, & z=0\end{cases}$$

is not analytic at $z=0$ though C-R equations are satisfied at $z = 0$.

22. State and prove Cauchy's Residue theorem.

23. State and prove Taylor's Theorem.

24. Evaluate $\int_0^{2\pi}\dfrac{d\theta}{2+\cos\theta}$.

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