MAY 2023 **U/ID 32518/UCM6C**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Evaluate $\underset{z \to 0}{Lt} \frac{z}{\bar{z}}$.

2. Find $f'(z)$ when $f(z) = \left(1 - 4z^2\right)^3$.

3. Define bilinear transformation.

4. Define conformal mapping.

5. Evaluate $\int_C \frac{z+2}{z} dz$ where C is the circle $z = 2e^{i\theta} (0 \le \theta \le 2\pi)$.

6. Find the antiderivate of $\int_1^3 (z-2)^3 dz$.

7. Define convergence of a series.

8. Define a zero of order m at z_0.

9. Define essential singularity.

10. If $z = e^{i\theta}$ the find the value of $\frac{1}{2i}\left(z - \frac{1}{z}\right)$.

11. Prove that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$ is convergent.

12. State argument principle.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Examine whether the function $f(z) = z^2$ is differentiable or not.

14. Discuss the transformation $w = \frac{1}{z}$.

15. State and prove Cauchy's integral formula.

16. Expand $\frac{z-1}{z^2}$ about $z = 1$ by using Taylor's series.

17. Find the residue of $\frac{z+1}{z^2(z-2)}$.

18. State and prove Rouche's theorem.

19. Prove that the zeros of an analytic function are isolated.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

20. Obtain the sufficient condition for differentiability of a function.

21. Discuss the transformation $w = \sin z$.

22. State and prove maximum modules theorem.

23. State and prove Laurent's theorem.

24. Show that $\int_0^{2\pi} \frac{d\theta}{1+a\sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}$ $(-1 < a < 1)$.

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