MAY 2024 **U/ID 32518/UCM6C**

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer any TEN questions.

1. Evaluate $\lim_{z \to 2i} \left(2x + iy^2\right)^2$.

2. State Cauchy Riemann equations.

3. Find the constants a and b so that the functions $f(z) = a(x^2 - y^2) + ibxy + c$ is differentiable at every point.

4. Find the transformation of the points $-1, 0, 1$ into the points $-i, 1, i$, respectively.

5. Find the fixed points of the mapping $W = \frac{1}{z}$.

6. Define contour.

7. Evaluate $\int_c xdz$ where C is the circle $|z| = 9$.

8. State Liouville's theorem.

9. Find the limit of the sequence $\left\{\frac{1}{n^3} + i\right\}_{n=1}^{\infty}$.

10. Expand $f(z)=\dfrac{z-1}{z+1}$ as a Taylor series about the point $z=0$.

11. Find the residue of $\dfrac{1}{z+z^2}$ at $z=0$.

12. Find the zero's of the function $f(z)=\dfrac{(z+1)(z-2)^3}{z+5}$.

PART B — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

13. Show that the function $f(z)=\bar{z}$ is nowhere differentiable.

14. Find the value of z for which $f(z)=\tan^2 z$ fails to be analytic.

15. Find the critical point of the transformation $w^2=(z-\alpha)(z-\beta)$.

16. Find the image of the circle $|z-1|=1$ under the transformation $w=\dfrac{1}{z}$.

17. Expand $\log\left(\dfrac{1+z}{1-z}\right)$ at $z=0$ using Taylor series.

18. Evaluate $\int_c \frac{(z+4)dz}{z^4+2z+5}$ where C is $|z+1-i|=2$.

19. Find the residue of $f(z)=\frac{1-e^{2z}}{z^4}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. If $u-v=(x-y)\left(x^2+4xy+y^2\right)$ and $f(z)=u+iv$ is an analytic function of $z=x+iy$, find $f(z)$ interms of z.

21. Find the image of the line $y=3x+1$ under the transformation $w=z^2$.

22. Discuss the transformation $w=\sin z$.

23. State and prove Laurent's theorem.

24. Evaluate $\int_C \frac{zdz}{(z-1)^2(z+1)}$ where C is the circle

(a) $|z|=\frac{1}{2}$

(b) $|z|=2$, using Cauchy residue theorem.

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