MAY 2025 **U/ID 32518/UCM6C**

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer any TEN questions.

1. Express $w = z\bar{z}$ in the form of $u(x, y) + iv(x, y)$.

2. Evaluate $\lim_{z \to 2} \frac{z^2 - 4}{z - 2}$.

3. Define differentiable function.

4. Find the critical point $w = 1/z$.

5. Find the fixed point of $w = \frac{1}{z - 1}$.

6. Describe Riemann surface in double-valued function.

7. State Cauchy's Theorem.

8. Evaluate $\int_c xdz$ where C is the circle $|z| = r$.

9. State Taylor's theorem.

10. Expand $\frac{1}{z[z-1]}$ as Laurent series about $z = 0$.

11. Define Pole.

12. Find and classify the singularities of $f(z) = \frac{z^2 - 2z + 3}{z - 2}$.

PART B — (5 × 5 = 25 marks)

Answer any FIVE questions.

13. Verify C-R equations for the function $f(z) = z^3$.

14. Find the analytic function $f(z) = u + iv$, if $u + v = \frac{\sin 2x}{(\cosh 2y - \cos 2x)}$.

15. Find the image of the circle $|z| = 1$ by the transformation $w = z + (3 + 2i)$.

16. Find the image of the circle $|z - 1| = 1$ under the transformation $w = 1/z$.

17. Show that $\int_c |z|^2 dz = -1 + i$ where C is the square with vertices $D(0,0), A(1,0), B(1,1)$ and $C(0, 1)$.

18. Expand $\log\left(\frac{1+z}{1-z}\right)$ at $z = 0$ using Taylor series.

19. Prove that $\int_C \frac{c^{2z}}{(z+1)^3} dz = \frac{4\pi i}{e^2}$ where C is $|z| = 3/2$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

20. State and prove necessary and sufficient conditions for analyticity in Cartesian Coordinate.

21. Show that the transformation $w = 1/z$ transforms circles and straight lines in the z-plane into circles and straight lines in the w-plane.

22. Derive Cauchy's integral formula.

23. State and prove Taylor's theorem.

24. Prove that $\int_0^{\infty} \frac{\cos x}{1 + x^2} \, dx = \pi/2e$.
